PROBLEMATUM QUORUNDAM MATHEMATICORUM,

(De Triangulis tam Rectangulis quam Obliquangulis,)

ANALYTICA SOLUTIO, ET CONSTRUCTIO.

Authore J. TWYSDEN.

CERTAIN MATHEMATICAL PROBLEMS,

(Concerning Triangles as well Oblique as Rectangled,)

ANALYTICALLY RESOLVED, AND EFFECTED,

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Problema I.

Datis trianguli plani rectanguli summa laterum (c) *&* *basi* (b) *in venire tum cathetum tum hypotenusam.*

Problem I.

In a plain rectangled triangle, (c) the sum of the hypotenuse, & perpendicular being given, together with (b) the base, to find the rest.

Puta factum sitque ,

(a) Cathetus, erit $c-a$ hypotenusa , & $cc-2ca$ plus aa ∞ $bb+aa$, & demptis utrinque, aa erit $cc-2ca$ ∞ bb , vel $2ca$ ∞ $cc-bb$ & $\frac{cc-bb}{2c}$ ∞ a.

Aliter ,

Sit à hypotenusa erit $c-a$ basis, & aa ∞ $cc-2ca+aa+bb$ & $\frac{cc+bb}{2c}$ ∞ a.

Canon.

Quadratum summæ laterum minutum qudrato basis, & per duplum laterum summam divisum exhibebit cathetum. Auctum vero quadrato basis, & per duplum laterum summam divisum exhibebit hypotenusam.

Canon.

If from the square of the sum of the sides, you take away the square of the base, and divide the residue by double the sum of the sides, the quotient shall be the quantity of the perpendicular. But if the square of the sum of the sides, be increased by the square of the base, and that sum divided by double the sum of the sides, the quotient shall be the hypotenuse.

☞ Fig. 1.

Example in numbers. Let the base be 3.

Let the sum of the sides be 9, *the Square* 81 , *the Square of the base* 9, 81 + 9 , *is* 90, *divided by* 18 , *shall give* 5 *the hypote-*

hypotenuse, or 81 − 9 *is* 72, *that divided by* 18, *shall give* 4, *the perpendicular, so the sides shall be* 3, 4, 5.

Fig. 2. Geometricè sic. Diametro A B ꝏ *c* describatur semicirculus. Cui inscribatur B *f* continuata infinite, & sit B *f* ꝏ *b* erit A *f* cui æquatur *f i*, √q. *c c* − *b b*. continuetur *f a* in *l*, ita ut *f l* sit æqualis 2 *c* inter quam ut prima, & *f i* ut secunda inveniatur, *f m* tertia, quæ æquabitur catheto quæsito, nam *f i* q. ꝏ *f a* q. ꝏ *c c* − *b b* dividitur per *l f* ꝏ 2 *c*. & sit *f m* quotiens geometricus. Nam *f l*. *f i*. *f m* sunt continuè proportionales; per 8 El. sexti *Eucl.* ergo $\frac{fi\,q.}{fl}$ producit *f m*, per 4 sexti *Eucl.*

Geometrically thus. Upon A B made equal to c, *the sum of the hypotenuse, and cathetus, describe a semicircle, in which, inscribe* B f, *from the term* B *equal to* b *the base given, and continue it infinitely, so shall A f, to which make* f i *equal, be the root square of* c c − b b, *continue* f a *in* l, *so that* f l *be made equal to* 2 c, *between this as the first, and* f i *the second, find* f m *the third in continual proportion, it shall be equal to the perpendicular sought, for* f i *q.* ꝏ f a *q.* ꝏ c c − b b *is divided by* f l ꝏ 2 c, *and* f m *is the geometrical quotient, for* f l. f i. f m *are continually proportional, by the 8th. of 6* Eucl. *Therefore* $\frac{fi\,q.}{fl}$ *produces* f m; *by the 4th. of the 6* Euclid.

PROBLEMA II.

In triangulo rectangulo datis p, *perpendiculo ab angulo recto in hypotenusam dimisso*, & b *differentia segmentorum hypotenusæ, invenire triangulum.*

PROBLEM II.

In a right angled triangle *p*, the perpendicular, let fall from the right angle upon the hypotenuse, and *b* the difference of the segments of the hypotenuse are given, to find the triangle.

Sit â minus segmentum, *b* + *a* erit majus, & *b* + *a* in *a*, hoc est, *b a* + *a a* ꝏ *p p*. Ergo

Canon. $\surd\ \frac{1}{4}bb + pp : \ \pm \frac{1}{2} b$ ꝏ *a*.

Potest

Potest Problema sic aliter proponi. Data media trium quantitatum continue proportionalium cum differentia extremarum invenire reliquas.

Geometrice sic. Perficitur super diametro E F infinita erigatur ad rectos *m* I æqualis *p* datæ & mensuretur *m* H æqualis $\frac{1}{2}$ *b* erit H I $\sqrt{\frac{1}{4}bb+pp}$ super hâc ut semidiametro scribatur semicirculus, & observatur Canon *Algebricus*. Nam E *m* est $\sqrt{\frac{1}{4}bb+pp}:+\frac{1}{2}b$, & *m* F est $\sqrt{\frac{1}{4}bb+pp}:-\frac{1}{2}b$. Nam E *m*. *m* I. *m* F sunt $\div$ propter similitudinem triangulorum, E I *m*. *m* I F.

The probleme may be thus otherwise propounded. In three quantities in continual proportion, the middle term is given, and the difference of the extremes. To find the rest.

Geometrically thus. Upon the diameter E F, *produced infinitely erect* m *I at right angles, equal to* p *the perpendicular given, and measure off* m H, *equal to* $\frac{1}{2}$ b, *draw* H *I which shall be* $\sqrt{\frac{1}{4}bb+pp}$ *by the* 47 *of the* 1 *of* Eucl. *upon that as semidiameter describe a semicircle, and the analytical Canon is observed. For* E m *is* $\sqrt{\frac{1}{4}bb+pp}:+\frac{1}{2}$ b, *and* m *F is* $\sqrt{\frac{1}{4}bb+pp}:-\frac{1}{2}$ b. *For* E m, m *I*. m *F* $\div$ *by reason of the similitude of the triangles* E *I* m. m *I F*. Fig. 3.

THEOREMA.

SI quadratum perpendiculi augeatur quadrante quadrati differentiæ datæ. Aggregati radix quadrata aucta dimidio differentiæ datæ erit E *m* majus segmentum. Minuta vero dimidio differentiæ erit *m* F minus segmentum, & inventum est triangulum.

THEOREME.

IF to the square of the perpendicular, you adde a quarter of the square of the difference given, and from the sum extract the square root, that root increased by half the difference, shall be equal to E m *the greater segment, but diminished by half the difference, shall be equal to* m *F the lesser segment. And all the parts of the triangle are known.*

PROBLEMA III.

Fig. 4. *Data media c trium quantitatum* $\because$ a, c, d, *una cum* b, *differentia qua major terminus excedit duplum minoris invenire terminos. Est octavam* Oughtredi, *in Clave aliter propositum, & resolutum.*

Data *b. c. d.*

Quæritur minor terminus, *a.*

PROBLEM III.

In three terms *a, c, d,* $\because$ *c* the middle term is given, with *b*, the difference between the greater term, and double the lesse. The terms are required.

Given b. c. d.

Sought the lesser term a.

Puta factum quod requiritur. Sitque *a*, minor extrema, erit $b + 2a$ major, & $ba + 2aa \propto cc$. vel $2aa \propto cc - ba$. Ergo $\sqrt{\frac{1}{4}bb + 2cc} : -\frac{1}{2}b \propto 2a$, scilicet duplo minoris termini. Vel quod idem est, $aa \propto \frac{cc - ba}{2}$, & $\sqrt{\frac{1}{16}bb + \frac{cc}{2}} : -\frac{1}{4}b. \propto a.$

THEOREMA.

DUplo quadrato mediæ datæ addatur quarta pars quadrati differentiæ datæ. Hujus summæ radix quadrata minuta dimidio differentiæ erit dupla minoris termini.

Fig. 5. Geometrice. Ad punctum *c* rectæ *bc* mensuretur $co \propto c$ datæ, cui æqualis statuatur *om* ad rectos: erit $cm \propto \sqrt{2cc}$. cui æquatur *cf* ducta à termino *c* ad rectos fac $ci \propto \frac{1}{2}b$ erit *if* æqualis $\sqrt{\frac{1}{4}bb + cc}$, cui æquetur *ib* subduc *ih* æqualis $\frac{1}{2}b$, erit $bh \propto 2a$, & $hc \propto$ differentiæ, super *bc* igitur diametro describatur semicirculus *blc*, in cujus circumferentia accomodetur $bl \propto c$, & dividatur *bh* bifariam in *n*, erit *bl*

THEOREME.

I*F to double the middle term squared, you adde a quarter of the square of the difference: the square root of this sum being diminished by half the difference, shall leave the lesser extreme sought.*

Geometrically. At the point c *of a right line* cb, *measure* co *equal to* c, *the middle term given, to which, make* om *at right angles equal. Then shall* cm *be the* $\sqrt{2cc}$, *to which* cf *is by construction equal & perpendicular to* bc, *make* ci *equal to half* b, *then shall* if *be* $\sqrt{\frac{1}{4}bb + cc}$, *to which, make* ib *equal: from* ib *subduct* ih *equal to* $\frac{1}{2}$ *the difference*, bh *shall be equal to double the lesser term sought, and* hc *shall* be

b l media, *b n* minor, & *b c* major trium quantitatum ∺, nam triangula *b l n*, *b l c*, sunt similia, ergo *c b. b l. b n* sunt ∺ & observatur præscriptum Theorematis.

be equal to the difference given. Upon b c, *as a diameter, describe a semicircle* b l c, *into which fit* b l ∞ c, *and divide* b h *into two equal parts*, b l *shall be the middle*, c b *and* b n *the two extremes, in continual proportion. For the triangles* b l n, b l c *are alike.*

PROBLEMA IV.

In triangulis duobus rectangulis dantur summa basium, & utrinsque cathetis ea conditione, ut angulus ad F *sit rectus. Quæruntur bases.*

PROBLEM IV.

Fig. 6.

In two right angled triangles, the sum of the two bases, each perpendicular, and a right angle at F are given. The bases are sought.

Dantur *b*, *c*, *d*, & angulus ad F rectus.

$aa . \infty aa$

$ee . \infty cc + aa - 2ca$

$gg . \infty cc + \overline{dd + bb - 2bd}$

$hh . \infty bb + aa$

$kk . \infty dd + ee$ (id est) $+ cc + aa - 2ca$

$gg . \infty (hh + kk)$ vel $bb + aa + dd + cc + aa - 2ca.$

$gg . \infty bb + aa + dd + cc + aa - 2ca.$

$gg . \infty cc + dd + bb - 2bd$, ergo hæ duæ species æquantur inter se, *viz.*

$cc + bb + dd - 2bd \infty bb + dd + cc + 2aa - 2ca.$

Et sublatis utrinque æqualibus.

$2aa - 2ca + 2bd \infty 0\,0$. Ergo mutatis signis $2aa \infty 2ca - 2bd$, & $aa \infty ca - bd$, & resoluta æquatione

$\frac{1}{2}c \pm \sqrt{\frac{1}{4}cc - bd} : \infty \frac{a}{e}$

THEOREMA

$\frac{1}{2}c \pm \sqrt{\frac{1}{4}cc - bd} : \infty \frac{c}{a}$

In verbis,

EX quadrato dimidii summæ basium tolle planum ex uno cathetorum ducto in alterum. Residui

In words,

OUt of the square of the sum of both the bases; take the plain made by one of the perpendi-

Fig. 6. Residui radix quadrata aucta dimidio summæ basium, erit basis trianguli majoris. Sed dimidium summæ basium minutum radice quadrata dicti residui erit basis trianguli minoris.

Geometrica effectio patet in figura. Est enim F A summa cathetorum,& quadratum BC æquatior plano F B A(hoc est *b d.*) E B est semis. (*c*)B E D est semic.B D ∞ BC,ergo D E est $\sqrt{q.\,\frac{1}{4}cc. - bd}$: D E. ∞ E G, ergo B G est $\frac{1}{2}$ c + $\sqrt{\frac{1}{4}cc - bd}$ H G est $\frac{1}{2}$ c − $\sqrt{\frac{1}{4}cc. - bd}$ quod requirit Theorema.

PROBLEMA V.

ANno 1644. *Johannes Pellius* Coritano Regnus *Anglus*, Matheseos in illustri *Amstelodamensium* Gymnasio Professor,chartulam quandam excudi curavit, & in varia loca dimisit continentem Theorema, quoddam cujus medio *Cristiani Severini*, *Longomontani*, *Cimbri*, *&c.* Librum de absoluta circuli mensura solide, & nervose refutavit, uti fusius in prædicti D. *Pellii* libello postea contra *Longomontanum* divulgato apparet. Hujus chartulæ prius impressæ exemplar unicum ad me misit D. *Guilielmus Beecher* Eques Auratus

pendiculars, multiplyed by the other. The square root of the residue, being increased by half the sum of the sides, shall be the base of the greater triangle: but half the sum of the sides diminished by the root of the said residue, shall be the base of the lesser triangle.

The effection is evident in the figure. For F A is the sum of the perpendiculars. And the square of B C is equal to the plain F B A. ∞ b d, *B E is half* (c)*B E D is a semicircle B D is equal to B C, therefore D E is* $\sqrt{\frac{1}{4}cc - bd}$ *D E* ∞ *E G, therefore B G is* $\frac{1}{2}$ c + $\sqrt{\frac{1}{4}cc - bd}$: *and H G is* $\frac{1}{2}$ c − $\sqrt{\frac{1}{4}cc - bd}$ *as the Theoreme requires.*

PROBLEM V.

IN *the yeer* 1644. *Mr.* John Pell *Professor of the Mathematicks in* Amsterdam *caused a certain paper to be printed, and dispersed abroad conteining a Theoreme, by help of which he hath both solidly, and substantially confuted* Longomontanus *his Book of the absolute measure of a circle, as may appear more largely in a Book since published by* Mr. Pell *against* Longomontanus. *One of those first papers, Sr.* William Beecher *then living at* Roven, *sent me to* Paris, *to whom I returned my answer after some dayes, whither it miscarried*

Auratus meamque postulavit sententiam, & Theorematis demonstrationem. Nonnullis ab accepta chartula diebus solutionem, & demonstrationem analyticam à *Parisiis* ad illum tunc Rothomagi degentem misi. Utrum vero ei in manus venerit ignoro. Erat autem Theorema ut sequitur.

ried or no, I know not. The Theoreme was as followeth. Fig. 6.

Tangens cujuslibet arcus minoris quam 45 g. 00 m. ducatur in duplum quadratum radii; à quadrato radii auferatur tangentis quadratum illud productum dividatur per hoc residuum: Quotus erit tangens arcus dupli.

Let the tangent of any arke lesse then 45 deg. 00 m. be multiplyed by double the square of the radius, from the square of the radius, take the square of the tangent. Let the first product be divided by this residue, the quotient shall be the tangent of the double ark.

Ego ad formam Problematis reduxi.

I reduced it into the form of the following Probleme.

Datis trianguli rectanguli basi (*r*) perpendiculi segmento angulo recto contermino (*t*), & angulo ad A bifariam secto invenire perpendiculum, & totum triangulum.

In a rightangled triangle, there is given the base (r,) *tht segment of the perpendicular conterminous to the right angle* (t,) *with the angle at A bisected, to find the perpendicular and the whole triangle.*

A E, est $\sqrt{q.}\ rr + aa$. per 47. 1 *Euclid.* Fig. 7.

Data. *r.* & *t.* Quæritur *a.* Quia per tertium sexti *Euclidis.*

$r.\ t. :: \sqrt{q.}\ rr + aa.\ a - t$ erit

$ar - tr. \infty \sqrt{q.}\ rrtt + ttaa$, ergo eorum quadrata erunt æqualia.

$rraa + rrtt - 2rrta \infty rrtt + ttaa$, vel subductis æqualibus.

$rraa - 2rrta \infty ttaa$, & dividendo,

$rra - 2rrt \infty tta$, vel transponendo terminos.

$2rrt \infty rra - tta$. Ergo

$rr - tt.\ 2rr :: t.\ a$, & propterea ex $\frac{2rrt}{rr - t}$ orietur *a.*

 Quod

Quod est ipsissimum Theorema D. *Pellii*. Posita enim basi trianguli pro radio erit *t*, tangens arcus simpli, & à tangens arcus dupli. Ergo si tangens cujuslibet arcus minoris quam 45 gr. 00 min. &c.

Fig. 7.

DETERMINATIO.

Hinc patet quod segmentum perpendiculi (hoc est tangens arcus simpli) non debet radium excedere (hoc est tangentem arcus 45 gr. 00 min.) aliàs enim subductio nequit fieri quod requirit Theorema.

Fig. 8. Geometricè sic. Super E G circuli radio ut diametro describatur semicirculus: mensuretur E C ∞ E T tangenti datæ, erit G Cq. ∞ *r r — t t*, cui æqualis statuatur A B, B *m* q. vero sit æqualis, lateri seu radici 2 E G q. hoc est 2 *r r*. Inter A B, & B *m*, hoc est, inter √q. *r r — tt*, & √q. 2 *r r* quæratur, tertia proportionalis quæ invenietur ∞ A E, & per 18 octavi *Eucl. r r — t t. 2 r r* : : A B. A E. Ergo, ut A B. AE : : *t. a.* Nam ut *r r — t t. 2 r r* : : *t. a.* Erigatur igitur à puncto B, perpendicularis B D ∞ E T, hoc est *t*. cui parallelæ ascendat infinita E F, & à puncto A per terminum D, ducatur A F erit E F ∞ (*a*) quæsitæ qua cognita compleatur triangulum Theoremati congruum.

PRO-

DETERMINATION.

From hence it appears, that the segment of the perpendicular, (to wit the tangent of the simple ark) must not exceed the radius (that is the tangent of 45 *gr.* 00 *m.) for otherwise the subduction cannot be made as the Theoreme requires.*

Geometrically thus. Upon E G *the Radius of your circle, as a diameter describe a circle. Set off* E C ∞ E T *the tangent given.* G Cq. *shall be equal to* r r — t t *to which make A B equal. And let B* m *q. be equal to 2 G E q. that is,* 2 r r. *Then between A B, and B* m, *that is, between* √q. r r — t t, *and* √q. 2 r r *find the third proportional, which let be A E. by the* 18 *of the 8th* Eucli. *A B q. shall be to B* m *q.* : : *A B. A E, that is,* r r — t t. 2 r r : : *A B. A E. for as the first is to the fourth, so shall the square of the first, be to the square of the second, in terms continually proportional, since it is therefore* r r — tt. 2 r r : : *A B. AE, and* r r — tt 2 r r : : t. a. *it shall be A B. A E* : : t. a. *therefore from the term B, erect a perpendicular, B D* ∞ E T, *that is, to* t, *to which draw* E F, *an infinite line parallel,*

rallel, and from the point A, by D, draw A F. E F shall be equal to (a,) which being found, finish the triangle agreeable to the Theoreme. Fig. 8.

PROBLEMA VI.

Data tangente arcus dupli quæratur tangens arcus simpli, hoc est data a quæratur t, *quia antea inventa est hæc æquatio* $tta + 2rrt \infty rra$. *erit* $tt. \infty rr - \frac{2rrt}{a}$. *Ergo* $\frac{\sqrt{rrrr+rr:}}{aa} - \frac{rr}{a} \infty t$.

PRaxis geometrica facilis est loco $\frac{rrrr}{aa}$ scribe ss. Hoc modo, ut $aa.\ rr :: rr.\ ss$. Ergo $\frac{aass}{aa} \infty \frac{rrrr}{aa}$. & $\sqrt{ss+rr}: - \frac{rr}{a} \infty t$.

PROBLEM VI.

The tangent of a double ark being given, if it be required, to find the tangent of the single ark, the equation will be $\frac{\sqrt{rrrr+rr:}}{aa} - \frac{rr}{a} \infty t$.

T*He geometrical effection is easie, in the place* $\frac{rrrr}{aa}$, *write* ss. *Thus* $aa.\ rr :: rr.\ ss$. *then* $\frac{aass}{aa} \infty \frac{rrrr}{aa}$, *and* $\sqrt{ss+rr}: - \frac{rr}{a} \infty t$.

PROBLEMA VII.

Dato triangulo quadratum inscribere.

PROBLEM VII.

To inscribe a square into a given triangle.

Sit basis trianguli b Fig. 9.
Perpendiculum p
Sit latus quadrati inscribendi a.
Ergo segmentum perpendiculi superius, erit $p - a$.
Et erit $p - a.\ a :: p.\ \frac{pa}{p-a} \infty b$.
Ergo $pa \infty bp - ba$, & $pa + ba \infty bp$. Ergo $p+b.\ b :: p.\ a$.

Praxis Geometrica est facillima, sit A C, $\infty\ p + b$. & C D $\infty\ b$, & sit B A $\infty\ p$. erit B E latus quæsitum $\infty\ a$.

The effection is very easie, & requires no more then in the three terms given, to find the fourth. Therefore, make A C ∞ b + p, *and C D.* ∞ b, *and B A* ∞ p.

Eodem modo circulus qui inſcribi poteſt maximus inveniatur, cujus diameter erit quadrati, latus diagonium.

Fig. 10. Hoc idem Problema ſic aliter abſolvitur.

∞ p. B E *ſhall be the ſide ſought.*

So may the greateſt poſſible circle be inſcribed, whoſe diagonium ſhall be equal to the diameter of the circle.

This Probleme is thus otherwiſe performed.

Sit (a) ſegmentum perpendiculi inter trianguli verticem, & latus quadrati inſcibendi, erit $p-a$ latus quadrati. Et erit,

$$\left.\begin{array}{l} p \,.\, c :: a \,.\, \frac{ca}{p} \\ p \,.\, d :: a \,.\, \frac{da}{p} \end{array}\right\} \begin{array}{l} \text{Secundo erit} \\ \text{Ergo } \frac{ca+da}{p} \infty p-a, \ \& \ ca+da \infty pp-pa. \end{array}$$

Et $ca + da + pa \infty pp$. Ergo $\frac{pp}{c+d+p} \infty a$. quâ ſublatâ à perpendiculo reſiduum, erit latus quadrati inſcribendi.

Canon. $\frac{pp}{c+d+p} \infty a.$

Fig. 11. Geometricè ſic. Ducatur ab æqualis $c+d+p$, & ſuper hâc ut diametro, deſcribatur ſemicirculus acb, menſuretur $bc \infty$ perpendiculo, cui æquatur be per ſtructuram, & à puncto (c) deſcendat perpendicularis (cd,) erit (bd) quotiens Geometricus, & æqualis (a,) nam $ab.\ cb.\ bd$ $\div$

Ducatur a termino (e)(ei) æqualis baſi trianguli, & ad (ba) normali, agantur denique (dg) (bf) parallelæ, & compleatur triangulum.

Geometrically thus. Make a b *equal to* c + d + p, *and upon it as a dinmeter, deſcribe a ſemicircle* a c b, *meaſure* b c *equal to the perpendicular, to which* b e *is equal by ſtructnre, from the point* (c) *let fall the perpendicular* (c d,) (b d) *is the Geometrical quotient equal to* (a,) *for* a b. c b. b d $\div$

Laſtly, from the point (e) *draw* (e i) *equal to the baſe of your triangle, and ſquare to* (a b) *draw* (d g) *and* (b f) *parallels, and complete the triangle.*

PRO-

PRO-

Fig. 12.

PROBLEMA VIII.

Dato triangulo rectangulum inscribere, cujus area sit ad aream trianguli in ratione possibili data.

r *ad* s. *Et sit area trianguli* m m.

PROBLEM VIII.

In a triangle given, to inscribe a rectangle, whose area shall be to the area of the triangle in any possible proportion, as

r to *s*, and the area of the triangle let be *m m*.

Puta factum sitque latus quæsitum *a*.

$$\left.\begin{array}{ll}\text{Primo} & p.c::p-a.\ \dfrac{pc-ca}{p}\\ \text{Secundo} & p.d::p-a.\ \dfrac{dp-da}{p}\end{array}\right\}\ \infty \text{ lateri rectanguli majori. Ducatur in } a.$$

$$\text{Erit } \frac{pca-caa+dpa-daa.}{p}\ mm::s.\ r.$$

$$\text{Ergo } smm \propto \frac{rpca-rcaa+rdap-rdaa.}{p} \text{ vel } \frac{psmm}{r}$$

$\infty\ c+d$, hoc est,

$$bpa-caa-daa, \text{ vel } \frac{psmm}{r}+baa \propto bpa, \ \&\ \text{tandem, } baa \propto bpa-\frac{psmm}{r} \text{ vel } aa \propto pa-\frac{psmm,}{br}$$

$$\text{Et } \sqrt{\tfrac{1}{4}pp-\frac{psmm}{rb}}:+\tfrac{1}{2}p \propto a.$$

The Æquation.

$$\sqrt{\tfrac{1}{4}pp-\frac{psmm}{rb}}:+\tfrac{1}{2}p. \propto a.$$

Determinatio. Absolutum datum non debet excedere quadratum semissis perpendiculi. Nam si superaverit rectangulum inventum erit areæ majoris quam inscribi poteſt.

Constructio Problematis. Primo reducatur *r b* planum ad quadratum, sit illud *n n.* Similiter

Determination. *The absolutum datum must not exceed the square of half the perpendicular, for otherwise the area found will be greater then can be inscribed.*

For the Geometrical construction. *First reduce* r b *to a square, let that be* n n. *In like* manner

Fig. 12. Similiter reducatur (ps) ad quadratum sit illud xx, & loco $\frac{psmm}{rb}$ scribatur $\frac{xxmm,}{nn}$ deinde fiat $nn.xx::mm.tt$, ergo $\frac{xxmm}{nn} \propto \frac{nntt,}{nn}$ & æquatio constructionis facillimæ, sic stabit

$\sqrt{\tfrac{1}{4}pp - tt} : + \tfrac{1}{2}p . \infty\, a.$

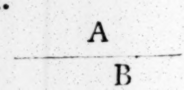
manner (p s,) *let that be* x x, *then in the place of* $\frac{psmm}{rb}$ *you will have* $\frac{xxmm}{nn}$ *then find the third proportion between* n n *and* x x. *As* n n.x x : : m m t t, *and then your Equation fit for construction will stand thus*

$\sqrt{\tfrac{1}{4}pp - tt} : + \tfrac{1}{2}p . \infty\, a.$

Problema IX.

Proposuit mihi vir ingenuus, & Philomathematicus, hanc quæstionem solvendam.

DAntur duæ lineæ sive numeri A & B, quarum summa (z.) æquatur differentiæ quadratorum. Summa vero quadratorum subducta ex quadrato summæ relinquet b planum.



Postquam paululum mecum ruminavi venit mihi in mentem Lemma sequens.

Lemma. Summa duorum quorumlibet numerorum unitate differentium, erit æqualis differentiæ quadratorum. Sin differant binario differentia quadratorum, erit dupla summæ, trinario tripla, &c.

Fig. 13. *Demonstratio.* A est major numerus & A + E. major, & E est

Problem IX.

An ingenuous person, and lover of the Mathematicks, propounded unto me this question.

T*Wo lines or numbers A and* B *are given, whose sum* (z) *is equal to the difference of their squares. But the sum of their squares being taken out of the square of the sum, the residue shall be equal to* b *planum.*

After I had a while thought upon, it there came into my mind this Lemma.

Lemma. *The sum of any two numbers differing by an unite, shall be equal to the difference of their squares. If their difference be two. Then shall the difference of the squares be double to the sum, &c.*

Demonstration. *Let A be the lesser number* , *and* A + E *the*

est 1. Differentia quadratorum erit 2 A E + E E. ꝏ z. Nam A in E. vel A in 1. hoc est A est A, est summa minoris numeri vel unius portionis. A E + E E, hoc est, A in 1 + 1. est summa majoris numeri unitate tantum excedentis minorem, ergo 2 A + 1 est summa utriusque numeri quod erat demonstrandum.

In numeris. Sit A + E. ꝏ 5. A − E ꝏ 4. 2 A E + E E, hoc est, 8 + 1 ꝏ 9. summæ numeri utriusque.

the greater. And let E *be an unite. The difference between the squares of A, and A +* E. *is* 2 *A* E + E E. *But* 2 *A* E + E E *is equal to* z. *For A* E *that is A, because* E *is an unite, is the lesser number, and A* E + E E, *that A* + 1 *is the greater number, therefore* 2 *A* E + E E *is the sum of both numbers equal to* z. *Which was to be demonstrated.* Fig. 13

Sit jam major linea a. Minor erit $a + 1$.

Summa $2a + 1$ ꝏ differentiæ quadratorum per Lemma præcedens.

$4aa - 4a + 1$ ꝏ Zq. Quadratum summæ.

$2aa - 2a + 1$ ꝏ Z summa quadratorum utriusque numeri.

Differentia. $2aa - 2a$ ꝏ b plano. Ergo $2aa.$ ꝏ b pl. $+ 2a.$ vice b pl. scribe $(bb.)$ Ergo $2aa.$ ꝏ $bb + 2a.$

Et $\sqrt{2bb + 1}: \begin{matrix} + \\ - \end{matrix} 1.$ ꝏ $\begin{matrix}\text{Duplo majoris lineæ.} \\ \text{Duplo minoris lineæ.}\end{matrix}$

Theorema. Si duplo residuo dato addatur 1. Hujus aggregati radix quadrata aucta unitate erit dupla majoris lineæ, minuta vero unitate erit dupla minoris.

Problema X.

In triangulo rectangulo dantur (b) *differentia hypotenusæ à crure minore, &* (p) *perpendiculo. Quæritur triangulum.*

Problem X.

In a right angled triangle there are given b, the difference between the hypotenuse, & the lesser leg or cathetus, together with (p) the perpendicular. The triangle is sought. Fig. 14.

Puta factum, & sit (a) pars quæsita, reliqua consonantibus notata dantur. Imo $b + a.\ p :: p.\ \frac{pp}{b+a}$ ꝏ a.

Ergo pp ꝏ $aa + ba$, & $\sqrt{\tfrac{1}{4}bb + pp}: - \tfrac{1}{2}b.$ ꝏ a.

Canon

Fig. 14. *Canon.* Quadrato perpendiculi, adde quartam partem differentiæ quadratæ aggregati radix quadrata, minuta dimidio differentiæ erit segmentum quæsitum.

Canon. *To the square of the perpendicular, adde a fourth part of the square of the difference, the square root of this aggregate shall exceed the segment sought, by half the difference given.*

Geometrice. Fiat $s\,t \propto \frac{1}{2}b$, & $t\,q. \propto p$. & sit angulus ad (t) rectus $s\,q$ erit $\sqrt{\tfrac{1}{4}bb + pp:}$ dematur $q\,m \propto \frac{1}{2}b$, erit $s\,m. \propto a$.

Geometrically. Make (s t) $\propto \frac{1}{2}$b, *and* t q $\propto$ (p), *and let the angle at* (a) *be right*, $(s\,q)$ *shall be* $\sqrt{\tfrac{1}{4}bb + pp:}$ *take away* q m $\propto$ *to half* b, s m *shall be equal to* a.

Problema XI.

Fig. 15. *Inscribere in circulo rectam (f) diametro minorem: ita ut si producatur infinite occurrat diametro productæ in puncto* (m) *dato.*

Problem XI.

To inscribe in a circle the right line f, *which must be less then the diameter, so that, if it be infinitely continued, it shall occurre with the diameter in the given point* m.

Data

Punctum *m*. Recta *f*.

Given

The point m. *The right line* f.

QUæritur portio lineæ *f* productæ à peripheriâ ad punctum *m*.

SOught. *The portion of the line* f, *continued from the periphery to the point* m.

Puta factum. Sit portio quæsita *a*. Per demonstrata à *Pitisco* ad Axioma quartum Triangulorum Planorum erit,

$f + a.\ b + c :: b - c.\ a.$ Ergo

$\frac{bb - cc}{f + a} \propto a.$ ergo $bb - cc \propto fa + aa.$ & $aa \propto bb - cc - fa.$

Et $\sqrt{\tfrac{1}{4}ff + bb - cc:} - \tfrac{1}{2}f. \propto a.$ quia vero $cc. \propto dd + \tfrac{1}{4}ff.$

Erit $\sqrt{\tfrac{1}{4}ff + bb - dd - \tfrac{1}{4}ff:} - \tfrac{1}{2}f. \propto a.$ vel $\sqrt{bb - dd:} \propto a + \tfrac{1}{2}f.$

Canon. $\sqrt{bb - dd:} \propto a + \tfrac{1}{2}f.$

Constructio patet in ipsâ figurâ. Est enim $\sqrt{bb - dd:} \propto a + \tfrac{1}{2}f.$ per 47. 1 *Eucli.*

The construction is apparent in the figure, for by 47. 1 Eucl. $\sqrt{bb - dd.} \propto a + \tfrac{1}{2}f.$

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Fig. 16.

PROBLEMA XII.

Ex dato rectangulo (y z) *à puncto* (t) *dato triangulum exterius abscindere æquale trapezio superiori* (h h) *dato.*

PROBLEM XII.

From the given rectangle (*y z*,) and from the known point (*t*) to cut off the exterior triangle equal to the upper trapezium.

Puta factum, & sit (*a*) basis trianguli majoris. Erit $a+c.b :: a. \frac{b\,a}{a+c} \infty$ istius trianguli catheto, & $\frac{b\,a\,a}{a+c} \infty 2\,d\,b$, vel $b\,a\,a, \infty 2\,d\,b\,a + 2\,d\,b\,c$, vel $a\,a \infty 2\,d\,a + 2\,d\,c$, &

THEOREMA.

$$\sqrt{d\,d+2\,d\,d\,c:} + d. \infty a.$$

Geometrice. Fiat A $c \infty 2\,d$, & $c\,e \infty$ C, ergo $c\,g. \infty \sqrt{q.}\,2\,d\,c$, & $z\,g$ vel $z\,d \infty \sqrt{q.}\,d\,d+2\,d\,c$. fiat $d\,f \infty d$, erit $z\,f$ quantitas quæsita.

Geometrically. Make A c ∞ 2 d, *and* c e. ∞ *to* C, *therefore* c g *is* $\sqrt{q.}$ 2 d c, *and* z g, *or* z d $\infty \sqrt{d\,d+2\,d\,c}$ *make* d f ∞ *to* d, z f *shall be the quantity sought.*

Fig. 17.

PROBLEMA XIII.

Ex dato rectangulo (y z) *à puncto* (t) *dato triangulum abscindere æquale spatio* (h) *dato.*

PROBLEM XIII.

From the given rectangle (*y z*.) from the given point (*t*,) to cut off a triangle equal to a trapezium known.

Puta factum, & sit basis trianguli abscindendi (*a*)

Erit $a+d+n.$ hoc est $a+c.\,b :: a. \frac{b\,a}{a+c}$ æquale lateri alteri triangulo ignoti (*viz.*) Catheto. Sed $\frac{b\,a\,a}{a+c}$ æquatur duplo areæ trianguli, id est (2 *h*.) Ergo $\frac{b\,a\,a}{a+c} \infty 2\,h$, vel $b\,a\,a \infty 2\,h\,a + 2\,h\,c$, & $a\,a \infty \frac{2\,h\,a+2\,h\,c.}{b}$ Ergo

Canon $\sqrt{\frac{h\,h}{b\,b} + \frac{2\,h\,c}{b}} : + \frac{h.}{b} \infty a.$

Fig. 18. Ad constructionem hujus Problematis, reducatur (*h*) superficies ad quadratum quod perinde vocetur (*h h*,) & Æquatio sic stabit:

$$\sqrt{\frac{bbhh}{bb}+\frac{2bhc}{b}:}+\frac{bh}{b}\ \infty\ a.$$

inter *b b* & *h h*. inveniatur tertia proportionalis *ff*, erit $\frac{bbff}{bb}\ \infty\ \frac{bbhh}{bb}$ Fiat secundo *b. h : : h. f.* erit $\frac{2bfc}{b}\ \infty\ \frac{2bhc}{c}$ similiter $\frac{bf}{b}$ erit $\infty\ \frac{hh}{b}$ & sic æquatio integra constructioni apta sic stabit,

$$\sqrt{\frac{bbff}{bb}+\frac{2bfc}{b}:}+\frac{bf}{b}\ \infty\ a.$$

vel $\sqrt{ff+2fc:}+f.\ \infty\ a.$

Fig. 18. In hoc Schemate, sit *h* latus trapezii ad quadratum reducti, & sit *ff* tertium proportionale inventum. Fiat 2 *f* + 2 *c*, diameter circuli *d g*, erit $\sqrt{q.}\ ff+2cf$, cui si addatur *d m* ∞ *f*. erit *g m* trianguli quæsiti basis qua cognita linea recta, à puncto *t*. ad terminum istum ducta abscindet trapezio dato triangulum æquale.

Fig. 19. Problema præcedens potest generalius proponi hoc modo.

Posito D angulo recto, à puncto *t*, dato supra basim DE

For the construction of this Probleme, you must first reduce the trapezium (h) *to a square, which may be called* (h h,) & *the Equation will stand thus,*

$$\sqrt{\frac{hhhh}{bb}+\frac{2hhc}{b}:}+\frac{hh}{b}\ \infty\ a.$$

then between b b, *and* h h, *find* ff *the third proportional. Then* $\frac{bbff}{bb}\ \infty\ \frac{hhhh}{bb}$ *Secondly, make* b. h : : h. f. *then* $\frac{2bfc}{b}\ \infty\ \frac{2hhc}{c}$ *in like manner, because* b f ∞ h h. $\frac{bf}{b}$ *shall be equal to* $\frac{hh}{b}$, & *the whole Equation will stand thus,*

$$\sqrt{\frac{bbff}{bb}+\frac{2bfc}{b}:}+\frac{bf}{b}\ \infty\ a.$$

or $\sqrt{ff+2fc:}+f.\ \infty\ a.$

In the Scheme, let (h) *be the side of a square equal to the trapezium, and* ff *the third proportional, between* (b b) & (h h:) *make* 2 f + 2 c *the diameter of a circle,* d g *shall be* $\sqrt{q}$ ff + 2 c f, *to which if you adde* d m *equal to* f, g m *shall be the base, of your triangle, and a streight line drawn from* t, *to that base shall cut off a triangle equal to the trapezium.*

The preceding Probleme may be more generally propounded in this manner.

From a given point t, *the angle at* D *being right, by position*

Fig. 1

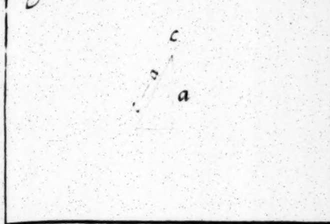


Fig. 2

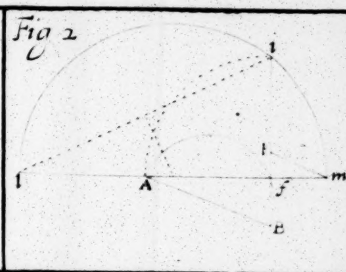


Fig. 3

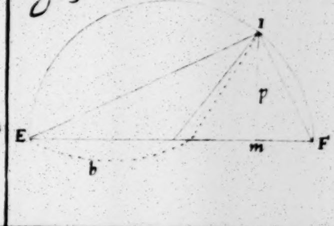


Fig. 4

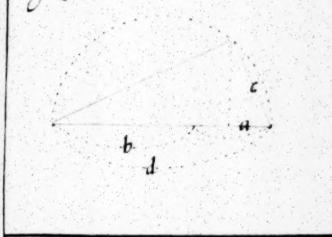


Fig. 5

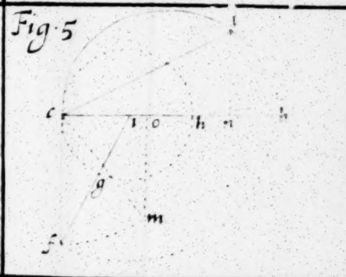


Fig. 6

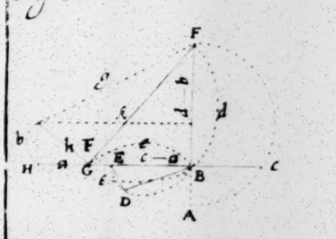


Fig. 7

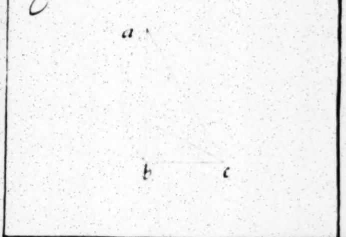


Fig. 8

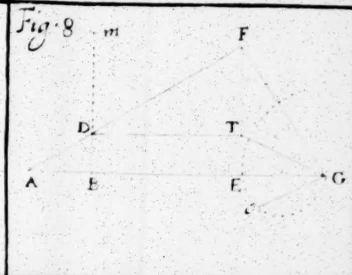


Fig. 9

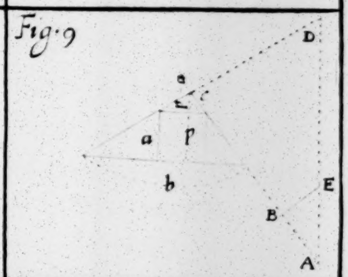


Fig. 10

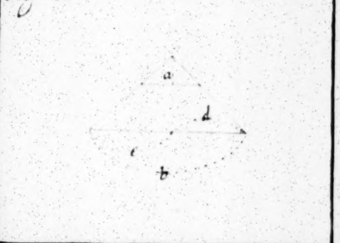


Fig. 11

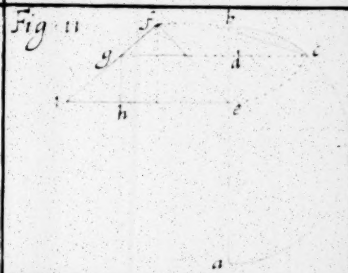


Fig. 12

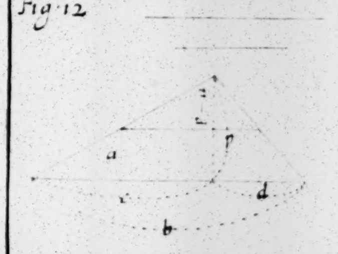


Fig. 13

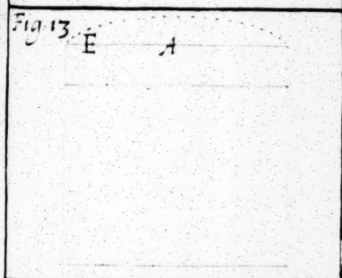


Fig. 14

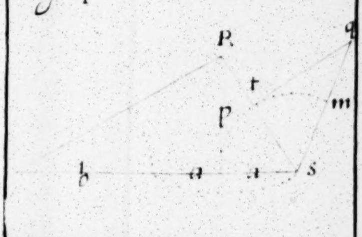


Fig. 15

Fig. 16

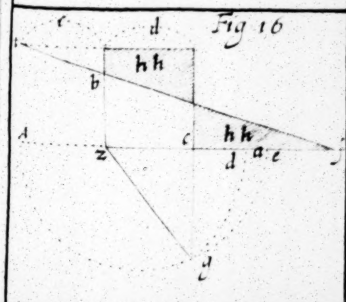
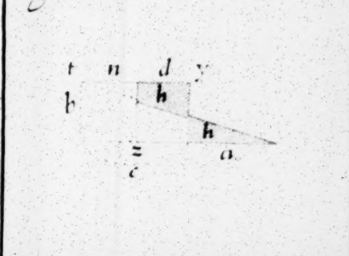


Fig. 17



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D E triangulum abſcindere æquale ſpatio dato, C.

tion, upon a baſe D E, *to cut off a triangle equal to any ſpace given, C.*

PROBLEMA XIV.

DAtam b lineam ita ſecare ut quadratum partis unius ſit equale plano ex altera parte cum externa data contentum.

Vel,

Data (ex tribus proportionalibus) una extremarum cum ſumma reliquarum invenire reliquas.

PROBLEM XIV.

T Is required to divide the given line b, *ſo that the ſquare of one of the parts, may be equal to the plain contained between the other part, and an external line given. Or,*

In three terms continually proportional, one of the extremes being given, and the ſumme of the other two, to find the terms.

Sit b linea data ſecanda. *Fig.* 20.
Sit d externa data.
Sit pars lineæ b ſecanda a.
Erit reliqua pars $b - a$.
Et $a\,a.$ ∞ $b\,d - d\,a.$ vel $\sqrt{\tfrac{1}{4}dd + bd:} - \tfrac{1}{2}d.$ ∞ $a.$

In verbis.

Si quadrato dimidii externæ datæ, addatur planum ex externa data in ſummam itidem datam. Aggregati radix quadrata minuta dimidio externæ datæ, erit media trium quantitatum $\div\!\!\div$

In words.

If to the ſquare of half the external line given, be added the plain made by the ſumme given, and the external line. The ſquare root of this aggregate leſſened by half the external line given, ſhall be equal to the middle term ſought.

Geometrice. Super A B. ∞ $b + d$ deſcribatur ſemicirculus, & ſit A C ∞ $b.$ à termino C erigatur perpendicularis C D. ducatur D E biſecans C B: erit D E vel E F $\sqrt{\tfrac{1}{4}dd + db:}$ & F C ſegmentum quæſitum, & erunt A F. F C. CB $\div\!\!\div$

Geometrically. Upon A B. ∞ b + d *deſcribe a ſemicircle, & let A C be equal to* b, *from C erect a perpendicular C D. draw D E biſecting C B. D E or E F ſhall be the* $\sqrt{\tfrac{1}{4}dd + db:}$ *and F C the ſegment ſought. So that A F. F C. C B* $\div\!\!\div$ *Fig.* 21.

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Problema XV.

PRopoſuit mihi (*Rothomagi*) Amicus quidam hanc quæſtionem ſolvendam, cujus voto ſatisfeci, & canonem addidi quo omnes hujus naturæ quæſtiones ſolvantur Quæſtio.

Datur rectangulum cujus area eſt $1345.\frac{5}{49}$ latitudo $\frac{2}{3} - 13$ longitudinis. Quæruntur latera.

Adhibeantur loco numerorum ſpecies.

Problem XV.

A Friend of mine (at Roven*) deſired of me the ſolution of this queſtion, whom I not only ſatisfyed, but gave him a Rule for the ſolution of all ſuch like Queſtions.*

A rectangle is given, whoſe area is $1345.\frac{5}{49}$, *the latitude* $\frac{2}{3} - 13$ *of the longitude. The ſides are ſought. Firſt the longitude.*

In the place of the numbers put Letters.

$$\text{Sint}\left\{\begin{array}{l} b \;.\; \infty\; 2 \\ c \;.\; \infty\; 3 \\ d \;.\; \infty\; 13 \\ f \;.\; \infty\; 1345.\frac{5}{49} \end{array}\right. \qquad \frac{b}{c} \infty \frac{2}{3}$$

Puta factum, & ſit (a) longitudo quæſita, erit latitudo $\frac{ba}{c} - d$, & $\frac{b}{c}aa - da \infty f$. vel $\frac{b}{c}aa \infty f + da$, & terminis omnibus in (c) ductis $baa \infty fc + dca$. Ergo $\frac{1}{2}dc + \sqrt{\frac{1}{4}ddcc + bfc} \infty ba$. Hinc Theorema, ſive

Canon. In omni rectangulo, ubi latitudo eſt longitudinis pars aliquo deficiens. Dico,

Si quarta pars quadrati defectus dati ducatur in quadratum denominatoris fracti, & huic facto adjiciatur area ducta in utroſque fractionis terminos. Hujus aggregati radix quadrata aucta dimidio defectus in fracti denominatorem ducti, exhibebit longitudinem totuplam quotupla eſt fracti numerator.

Canon. *In every rectangular figure, whoſe latitude is any aliquot part of the longitude deficient. I ſay,*

If a quarter of the ſquare of the defect given, be multiplyed into the ſquare of the fractions denominator, and to this product be added the area drawn into both the terms of the fraction. The root ſquare of this aggregate, increaſed by half the given defect, ſhall exhibit a longitude ſo much greater

numerator. Unde nec longitudo nec latitudo ignorabitur.

Experiamur in numeris juxta Canonem.

greater then the truth, as the numerator of the fractions consists of units. So that the true longitude and latitude cannot be unknown.

Let us examine by numbers according to the Canon.

$$\tfrac{1}{2}dc + \sqrt{\tfrac{1}{4}ddcc + bfc} : \propto ba.$$

$\frac{1}{2}dc. \propto \frac{39}{2}$
$\frac{1}{4}ddcc \propto \frac{1521}{4}$
$bfc. \propto \frac{295160}{49}$
$\left\} \frac{ddcc}{4} + bfc. \propto \frac{1656369}{196}\right.$ terminis (sc.) ad unum idemque nomen prius reductis. Hujus radix quadrata est $\frac{1287}{14}$ huic addi debent $\frac{39}{2}$ facit $\frac{3120}{28}$, id est, $111.\frac{3}{7}$ vel $\frac{780}{7} \propto ba$, cujus dimidium, quia (b) est (2) $\frac{780}{14}$ vel $55.\frac{5}{7}$ æqualis longitudini quæsitæ, & latitudo invenietur $24.\frac{1}{7}$. Nam $\frac{520}{14}$ est $37.\frac{2}{14}$. Sed $\frac{520}{14} - \frac{13}{1}$ hoc est minus, $\frac{338}{14}$ est $24.\frac{1}{7}$. Duc $55.\frac{5}{7}$ in $24.\frac{1}{7}$, hoc est $\frac{390}{7}$ in $\frac{169}{7}$ facit $\frac{65910}{49}$, vel $1345.\frac{5}{49}$ æqualis areæ datæ, ergo latera verè sunt inventa.

Problema XVI.

Requiritur secare $\sqrt{q}.125 + 5$ extrema, & media ratione.

Quia planum 125 provenit ex ductu 25 in 5. ergo media proportionalis inter 25 & 5. erit $\sqrt{q}$. 125.

Sit jam AB 25 talium partium qualium BC est 5. BD est media proportionalis inter AB, & BC, dico BD esse $\sqrt{q}$. 125, cui si adjiciatur DE $\propto$ BC. erit BE $\sqrt{q}$. 125 + 5. linea data secanda.

Problem XVI.

Fig. 22.

Tis required to cut $\sqrt{q}$. 125 + 5 in extreme, and mean proportion.

Because the plain 125 is produced by the multiplication of 25 into 5, therefore a mean proportional between 25 and 5 shall be the $\sqrt{q}$. 125.

Let AB be 25 such parts as BC is 5. BD is a mean proportional between AB & BC. I say therefore BD is $\sqrt{q}$. 125 to which, if you adde DE equal to BC. BE shall be $\sqrt{q}$. 125 + 5. The line given to be cut.

*Fig.*22. Sit a. major portio

$b \propto BE$ linea integra.

Erit $b - a$. minor portio, &

$b. a :: a. b - a$ ergo $aa \propto bb - ba$. &

$\sqrt{\tfrac{1}{4}bb + bb} : - \tfrac{1}{2}b. \propto a.$ majori portioni.

Geometrice, sit B F $\propto \tfrac{1}{2}$ BE erit F E $\sqrt{q}.\tfrac{1}{4}bb + bb$ fiat FG $\propto$ FB erit EG $\sqrt{\tfrac{1}{4}bb + bb}:$ $- \tfrac{1}{2}b$. fiat E H $\propto$ G E. Dico E B hoc est, $\sqrt{q}.\ 125. + 5$ esse sectam in H extrema, & media ratione Geometrice cujus major portio est E H, minor B H.

Geometrically. Make B F $\propto \tfrac{1}{2}$ BE. *F E shall be the root square* $\tfrac{1}{4}$ b b + b b, *make F G equal to F B. E G shall be* $\sqrt{q}.\tfrac{1}{4}bb + bb - \tfrac{1}{2}b$. *make* E H $\propto$ *to G E. I say, E B that is,* $\sqrt{q}.\ 125 + 5$ *is Geometrically cut in extreme and mean proportion, whose greater portion is* E H, *the lesser* H B.

Sed quia quæstio proponitur numerose. Numerose rem aggrediamur.

Sit $\sqrt{q}.\ 125 + 5$. secanda extrema, & media ratione.

Sit majus segmentum $1\sqrt{}$. Erit Ut $\sqrt{q}.\ 125 + 5.\ 1\sqrt{} :: 1\sqrt{}.\ \dfrac{1q.}{\sqrt{q}.\ 125 + 5.}$ Ergo $\dfrac{1q}{\sqrt{q}.125 + 5} + 1\sqrt{} \propto \sqrt{q}.\ 125 + 5.$ Et $\dfrac{1q}{\sqrt{q}.\ 125 + 5} \propto \sqrt{q}.\ 125 + 5 - 1\sqrt{}$, & $1q \propto 150 + \sqrt{q}.\ 12500 - \sqrt{q}.\ 125q. - 5\sqrt{}$. Hæc æquatio est jam solvenda.

But because the question is propounded in numbers, let us attempt it in numbers.

$\sqrt{q}.\ 125 + 5$ *is to be cut in extreme, and mean proportion.*

Let the greater segment be $1\sqrt{}$. *It shall be, As* $\sqrt{q}.\ 125 + 5.\ 1\sqrt{} :: 1\sqrt{}\ \dfrac{1q}{\sqrt{q}.\ 125 + 5.}$ *Therefore* $\dfrac{1q}{\sqrt{q}.\ 125 + 5} + 1\sqrt{}$ *equal* $\sqrt{q}.\ 125 + 5$. *And* $\dfrac{1q}{\sqrt{q}.\ 125 + 5} \propto \sqrt{q}.\ 125 + 5 - 1\sqrt{}$, *and* $1q. \propto 150 + \sqrt{q}.\ 12500 - \sqrt{q}.\ 125q. - 5\sqrt{}$. *This equation is now to be solved.*

Dimidium Radicum

$\sqrt{q}.\ 31\tfrac{1}{4}q. + 2\tfrac{1}{2}$

$\sqrt{q}.\ 31\tfrac{1}{4} + 2\tfrac{1}{2}$

Ejus quadratum est

Half the number of Roots is

$\sqrt{q}.\ 31\tfrac{1}{4}q. + 2\tfrac{1}{2}$

$\sqrt{q}.\ 31\tfrac{1}{4} + 2\tfrac{1}{2}$

The square of $\tfrac{1}{2}$ *the number of Roots is*

$31\tfrac{1}{4} + \sqrt{q}.\ 781\tfrac{1}{4} + 6\tfrac{1}{4}$, id est, $37\tfrac{1}{2} + \sqrt{q}.\ 781\tfrac{1}{4}$.

Idem

Fig. 22.

Idem hoc quadratum adnexum numero absoluto facit $37\frac{1}{2}+\sqrt{q}.781\frac{1}{4}+150+\sqrt{q}\,12500$. Hoc est $\sqrt{}\,187\frac{1}{2}+\sqrt{q}.781\frac{1}{4}+\sqrt{q}.12500$. Et quia duo surdi numeri sunt commensurabiles, & proportio quadratorum est $\frac{16}{1}$ erit ergo proportio radicum, $\frac{4}{1}$ multiplicanda igitur est minor $\sqrt{q}$. per 5, hoc est ducenda est $\sqrt{q}.781\frac{1}{4}$ in $\sqrt{q}.25$, & producetur $\sqrt{q}.19531\frac{1}{4}$ pro summa surdarum quantitatum. Jam igitur summa numeri absoluti & quadrati è dimidio radicum numero est $187\frac{1}{2}+\sqrt{q}.19531\frac{1}{4}$ hujus autem binomii $\sqrt{q}$ est $\sqrt{q}.156\frac{1}{4}$ plus $\sqrt{q}.31\frac{1}{4}$ vel $12\frac{1}{2}+\sqrt{q}.31\frac{1}{4}$, atque hæc radix minuta dimidio radicum numero, id est, $\sqrt{q}.31\frac{1}{4}+2\frac{1}{2}$ est valor $1\sqrt{}$ primo positæ. Sic igitur stabunt termini.

This square added to the absolute number, makes $37\frac{1}{2}+\sqrt{q}\,781\frac{1}{4}+150+\sqrt{q}.12500$, *that is,* $187\frac{1}{2}+\sqrt{q}.781\frac{1}{4}+\sqrt{q}.12500$. *And because these two surd numbers are commensurable, and the proportion of their squares, is as* $\frac{16}{1}$ *the proportion of their roots shall be* $\frac{4}{1}$. *Therefore the lesser* $\sqrt{q}$. *is to be multiplyed by* 5, *that is,* $\sqrt{q}.781\frac{1}{4}$ *in* $\sqrt{q}.25$, *the product will be* $\sqrt{q}.19531\frac{1}{4}$ *for the sum of the surd quantities. Now the sum of the absolute number, and the square of half the number of roots is* $187\frac{1}{2}+\sqrt{q}.19531\frac{1}{4}$. *The root square of this binome is* $\sqrt{q}.156\frac{1}{4}+\sqrt{q}.31\frac{1}{4}$, *or* $12\frac{1}{2}+\sqrt{q}.31\frac{1}{4}$, *& this root diminished by half the number of roots, that is,* $\sqrt{q}.31\frac{1}{4}+2\frac{1}{2}$ *is the value of that which at first was supposed* $1\sqrt{}$. *The terms will stand thus:*

$12\frac{1}{2}+\sqrt{q}.31\frac{1}{4}-\sqrt{q}.31\frac{1}{4}-2\frac{1}{2}$, id est $12\frac{1}{2}-2\frac{1}{2}$, id est 10.

Tota igit. lin. secanda est	$\sqrt{q}\,125+5$	*The whole line to be cut.*
Majus segmentum est	10	*The greatest segment.*
Minus segmentum est	$\sqrt{q}\,125-5$	*The lesser segment.*

Problema XVII.

Datâ (mm) area trianguli æquilateri invenire latera.

Problem XVII.

Fig. 23.

The area (*m m*) of an equilateral triangle being given to find the sides.

Esto p perpendiculum bisecans basim, & sit a semissis basis, ergo $2a$ erit basis integra, & $4aa \infty pp+aa$, ergo $3aa \infty pp$ & $\sqrt{q}.3aa \infty p$, sed $pa \infty mm$, ergo $\sqrt{q}.$

$3aa$

Fig. 23. $3\,aa$ in a, hoc est, $\sqrt{q}.\ 3\,aaaa \propto mm$, vel etiam $\sqrt{q}.\ aaaa \propto \sqrt{q}.\ \frac{1}{3}\,mmmm$, vel $aaaa \propto \frac{1}{3}\,mmmm$. Ergo $\frac{1}{3}\,mm.\ aa :: aa.\ mm$, vel denique $\sqrt{q}.\ \frac{1}{3}\,mm.\ a :: a.\ m$. Nam si quadrata sint proportionalia erunt, & radices quadratæ eorum proportionales. Ergo media proportionales inter m & $\sqrt{q}.\ \frac{1}{3}\,mm \propto a$.

Theorema. $aaaa \propto \dfrac{mmmm}{3}$

EX tertia parte areæ in se multiplicatæ educ radicem biquadratam quotiens exhibebit semissem lateris trianguli æquilateri.

EXtract the biquadratick root of the third part of the area biquadrated, the quotient shall give half one of the sides of the equilateral triangle.

Fig. 24. Geometrica praxis. Quia $\sqrt{q}.\ \frac{1}{3}\,mm.\ a :: a.\ m$. inveniatur media proportionalis inter m, & $\frac{1}{3}\,mm$.

Fiat $cd\ \frac{1}{3}\,m$, cui æquatur de, & sit $bd \propto m$, erit $cl\ \frac{1}{3}\,mm$ & dh quadratum æquale cl oblongo quadrata $bd.(m)\ hd :: hd.\ de\ (\frac{1}{3}\,m,)$ ergo dh æqualis $\sqrt{q}.\ \frac{1}{3}\,mm$. Nam be est diametrus circuli descripti super $bd \propto m$, & $de \propto \frac{1}{3}\,m$. Fiat $df \propto hd$, & diametro bf describatur semicirculus erit $di\ q \propto m$ in $\sqrt{q}.\ \frac{1}{3}\,mm$, & perinde æqualis semissi lateris cujuslibet incogniti. Fiat igitur $ng \propto 2\,di$, & compleatur triangulum.

Geometrically. Make c d $\frac{1}{3}$ m *to which, let* d e *be equal, and* b d *equal to* m, *the oblong* c l *shall be equal to* $\frac{1}{3}$ mm. *Therefore if upon* b e *you describe a semicircle,* h d *shall be equal to* $\sqrt{q}\ \frac{1}{3}$ m m, *because the square of it is equal to the oblong* c l $\propto \frac{1}{3}$ m m. *Make* d f *equal to* d h, *and upon* b f *as a diameter describe a semicircle;* d i *q. shall be equal to* m *in* $\sqrt{q}.\ \frac{1}{3}$ m m, *and therefore* d i *shall be equal to* (a) *half the side unknown, double* d i, *that is* n g, *shall be equal to the side of the equilateral triangle.*

In Fig. 24. No. 2 lineæ t t, *pars quarta erit* $\propto$ a, *vel dimidium erit* g n, *lateri trianguli integro.*

Aliter Geometrice. Quia $aaaa \propto \dfrac{mmmm}{3}$ fiat $3.\ mm :: mm.\ tt$: erit $\dfrac{3\,tt}{3} \propto \dfrac{mmmm}{3}$ vel $tt \propto \dfrac{mmmm}{3} \propto aaaa$, ergo

Otherwise Geometrically. Because $aaaa \propto \dfrac{mmmm}{3}$, *make* $3.\ mm :: mm.\ tt$. *and* $\dfrac{3\,tt}{3} \propto \dfrac{mmmm}{3}$ *therefore* $tt \propto \frac{1}{3}\,mmmm \propto aaaa$, *and* $t. \propto aa$,

ergo $t. \propto a\,a$. linea quadrato quadretur igitur linea inventa hoc est assumatur pars quarta erit æqualis a.

$\propto$ a a, *a line to a square. If therefore you square the line found, that is, take a fourth part of it shall be equal to* a *sought. I say, in the figure* No. 2 $\frac{1}{4}$ t. $\propto$ a, *or* $\frac{1}{2}$ t. $\propto$ *to the side of the triangle.*

PROBLEMA XVIII.

In triangulo rectangulo a x b *datis* a, b, *& recto ad centrum circuli invenire* x.

PROBLEM XVIII.

In a right angled triangle $a\,x\,b$, a and b are given, and the right angle at the center of the circle, to find x.

Fig. 25.

Vide Doctissimi Franc. à Schooten, *commentaria in lib.* 2. *Rei Geometricæ* Renat. des Cartes *pag.* 273-274 *quæ serius vidi post Probl. solutionem.*

Puta factum, & sit latus quæsitum x.

Erit, Ut $b.\ x + a :: x - a.\ \frac{xx - aa}{b}$ Ergo

Quadratum $\frac{xx - aa}{b} \propto (c)$ erit æquale $xx + aa$

Viz. $\frac{x^4 + a^4 - 2xxaa}{bb} \propto xx + aa$.

Et $x^4 + a^4 - 2xxaa \propto xxbb + aabb$

Et $x^4 - 2xxaa - xxbb \propto aabb - a^4$, vel per transpositionem terminorum.

$2xxaa + xxbb - x^4 \propto a^4 - aabb$

$aa + \frac{1}{2}bb$. Ergo Quadratum $aa + \frac{1}{2}bb$ est $a^4 + bbaa + \frac{b^4}{4}$

$aa + \frac{1}{2}bb \pm \sqrt{\frac{1}{4}b^4 + a^4 + bbaa - a^4 + bbaa} \propto xx$

$aa + \frac{1}{2}bb \pm \sqrt{\frac{1}{4}b^4 + 2aabb} \propto xx$, vel denique

$aa + \frac{1}{2}bb \pm b\sqrt{\frac{1}{4}bb + 2aa} \propto xx$

THEOREMA.

$aa + \frac{1}{2}bb \pm b\sqrt{\frac{1}{4}bb + 2aa} \propto xx$

Vel $\sqrt{aa + \frac{1}{2}bb + b\sqrt{\frac{1}{4}bb + 2aa}} \propto x$.

Praxis Geometrica est facillima, & patet in Schemate.

The Geometrical effection is very easie, and appeares in the Scheme.

*Fig.*26. Fiat $AB \propto b$, erit $BC \surd q. \frac{1}{4} bb$. Sit $BF q. \propto 2aa q$.
$BD \propto \frac{1}{4} b$ & $BE \propto b. \surd \frac{1}{4} bb$. Erit $EF \propto b \surd \frac{1}{4} bb + 2aa$,
cui in directum adjiciatur
$FG \propto \surd. 2aa + \frac{1}{4} bb$. Erit $EG \propto \surd\overline{aa + \frac{1}{4} bb + b. \surd \frac{1}{4} bb + 2aa.}$
$\propto x$, qua cognita compleatur triangulum Schemati congruum.

PROBLEMA XIX.

In triangulo plano rectangulo. Dato perpendiculo una cum aggregato basis, & duplæ hypotenusæ invenire ipsas, tum hypotenusam tum basim.

PROBLEM XIX.

In a right angled triangle there are given the perpendicular, the sum of the base & double the subtense. The subtense & base are sought.

Puta factum & sit basis quæsita.

*Fig.*27. Sit basis a. Erit

$$aa + bb \propto \frac{dd + aa - 2da}{4} \text{ vel}$$

$4aa + 4bb \propto dd + aa - 2da$, & demptis utrinque aa.

$3aa + 4bb \propto dd - 2da$, &

$3aa + 2da \propto dd - 4bb$ vel

$$aa + \frac{2da}{3} \propto \frac{dd - 4bb}{3} \;\&\; \surd\overline{\frac{4dd}{9} + \frac{dd - 4bb}{3}} : - \frac{d}{3} \propto a, \text{ vel}$$

$$\surd\overline{\frac{4dd - 12bb}{9}} : \text{ minus } \frac{d}{3} \propto a.$$

Praxis in numeris.

$d \propto 14$
$b \propto 3$

$dd \propto 196$
$bb \propto 9$

$4dd \propto 784$
$12bb \propto 108$

Differentia $\propto \frac{676}{9}$

Hujus

Hujus $\surd$q. $\frac{26}{3}$
Hinc tolle $\frac{d}{3}$ $\frac{14}{3}$

Restat $\frac{12}{3} \propto 4$

Ergo basis est	4	Tria latera	
Dupla hypot.	10		3
Hypotenusa	5		4
Et perpend.	3		5

THEOREMA.

$$\surd\, \frac{4dd - 12bb}{9} : \text{minus } \frac{d}{3} \propto a.$$

Praxis Geometrica.

Fig. 28.

Fiant $\left\{ \begin{array}{l} ce \propto d \\ cb \propto \frac{4}{9}d \\ ep \propto \frac{1}{3}d \\ ci \propto b \\ ek \propto \frac{4}{3}b \end{array} \right.$

Inde $ce\,(d)\; ek\,(\frac{4}{3}b) :: ci\,(b)\; il\,(f)$

Fiat $bm \propto f$.
& $en \propto em$ ergo
$\square\; cen \propto {}^{4}dd - \frac{4}{3}bb$ &
eo $\surd$q. ejusdem
Fiat $oq \propto \frac{1}{3}d \propto ep$
erit $eq \propto$ basi (a) &
$\frac{1}{2}cq \propto$ hypotenusæ.

Idem fieri poterit pro aggregato basis, & triplo quadruplo, quintuplo, &c, hypotenusæ. Pro triplo hypotenusæ æquatio sic stabit,

The same thing may be done where the sum of the base, and treble quadruple, quindruple, &c. of the hypotenuse are given. Where treble the hypotenuse is given the equation will be,

$$\surd\, \frac{9dd - 72bb}{64} : -\frac{d}{8} \propto a.$$

PRO-

Problema XX.

In triangulo plano rectangulo. Data hypotenusa una cum aggregato perpendiculi & duplo basis in... ire perpendiculum & basim.

Problem XX.

In a plain triangle. The hypotenuse and aggregate of the cathetus, and double the base being given, to find the rest.

Fig. 29. Sit factum. Erit $\frac{z-a}{2}$ ∞ basi, ergo $aa + \frac{zz + aa - 2za}{4}$, vel $\frac{4aa + zz + aa - 2za}{4}$ ∞ bb, vel

$5aa + zz - 2za$ ∞ $4bb$, vel
$5aa - 2za$ ∞ $4bb - zz$, vel
$2za - 5aa$ ∞ $zz - 4bb$ vel
$\frac{2z}{5}a - aa$ ∞ $\frac{zz - 4bb}{5}$ Ergo

Theorema $\frac{z}{5} \mp \sqrt{\frac{20bb - 4zz}{25}}$: ∞ a.

Idem fieri poterit pro aggregato perpendiculi & triplo (quadruplo quintuplo, &c.) pro triplo basis æquatio sic stabit

$\frac{z}{10} \sqrt{\frac{90bb - 9zz}{100}}$ ∞ a.

Problema XXI.

In quovis triangulo plano. Datis basi, area, & differentia laterum invenire triangulum.

Problem XXI.

In any plain triangle whatsoever. Having the base the area and difference of the sides, to find the triangle.

Fig. 30. Sit trianguli area æqualis gg, ergo $\frac{2gg}{b}$ ∞ perpendiculo.

Dantur
b. ∞ Basi
gg. ∞ Area
d. ∞ differ. Crurum
Quæritur latus minimum a.

Ut

Ut $b. d + 2a :: d. \frac{dd + 2da}{b} \infty o$. Hanc tolle ex b erit
$\frac{bb - dd - 2da.}{b} \infty 2e$. Et $\frac{bb - dd - 2da:}{2b} \infty e$.
hujus autem quadratum est
$\frac{bbbb - 2bbdd - 4bbda + dddd + 4dddda + 4ddaa,}{4bb}$
cui addatur quadratum perpendiculi
$\frac{2gg}{b}$ hoc est, $\frac{4gggg}{bb}$ sed prius reducatur sic $\frac{16gggg}{4bb}$
Eritque $\frac{16g^4 + b^4 + 2bbdd - 4bbda + d^4 + 4d^3a + 4ddaa}{4bb} \infty aa$.
Id est $16g^4 + b^4 + 2bbdd - 4bbda + d^4 + 4d^3a +$
$4ddaa \infty 4bbaa$, vel
$16g^4 + b^4 - 2bbdd + d^4 \infty 4bbaa - 4ddaa - 4d^3a$
$+ 4bbda$. Et hujus æquationis parte ultimâ diuisâ per
$4bb - 4dd$. Quotus erit $aa + da$. Ergo etiam erit
$aa + da \infty \frac{16g^4 + b^4 + d^4 - 2bbdd}{4bb - 4dd}$ Nam ut prior ita, &
posterior pars æquationis dividenda est per $4bb - 4dd$.
Ergo pro solutione Problematis.
$\sqrt{\frac{1}{4}dd + \frac{16g^4 + b^4 + d^4 - 2bbdd}{4bb - 4dd}} : - \frac{1}{2}d. \infty a$, vel re-
ducto $\frac{1}{4}dd$ ad idem nomen,
$\sqrt{\frac{bbdd - d^4 + 16g^4 + b^4 + d^4 - 2bbdd}{4bb - 4dd}} : - \frac{1}{2}d \infty a$.
Vel deletis æquivalentibus erit
sic $\sqrt{\frac{16g^4 + b^4 - bbdd}{4bb - 4dd}} : - \frac{1}{2}d \infty a$, vel denique
sic $\sqrt{\frac{4gggg}{bb - dd} + \frac{1}{4}bb} : - \frac{1}{2}d. \infty a$.

Theorema. $\sqrt{\frac{4gggg}{bb - dd} + \frac{1}{4}bb} : - \frac{1}{2}d. \infty a$.

Geometrice. Fiat C med. proport. inter $b + d$ (k l) & $-d$ (k m) ergo Cq. $\infty bb - dd$ ($\infty b + d$ in $b - d$) atque

Geometrically. Make C a mean proportional between b + d (k l) *and* b−d (k m) *therefore C quad.* ∞ b b − d d (∞ b + d *in* Fig. 31.

Fig. 31. que C (kp) est radix quadrata ejusdem. Itidem $2gg$ est $\sqrt{q}.\ 4gggg$, applicentur igitur $2gg$ (vel Hq,) ad (kp vel) C, hoc est, fiat C. H :: H. F. Ergo C in F ∞ $2gg$ (∞ Hq.) & F est quasi quotus ex hac applicatione. Quadratum igitur ex F (nimirum Fq.) ∞ $\frac{4gggg}{bb-dd}$ huic adde $\frac{1}{4}bb$, id est, fiat rs (ad angulos rectos) ∞ $\frac{1}{2}b$, & agatur (ks) igitur (ks) est $\sqrt{q}.\ \frac{4gggg}{bb-dd}$ plus $\frac{1}{4}bb$; ex quâ aufer $\frac{1}{2}d$ (∞ st) restabit kt ∞ a; & si addas (sx) ∞ $\frac{1}{2}d$ ad kt erit kx ∞ $d+a$, ex tribus igitur jam datis lateribus $b.\ a.\ a+d$, vel etiam kn, kt, kx fabricetur triangulum nAk Schemati congruum

d *in* b − d, *but* C *is the root square of it. So also,* 2gg *is* $\sqrt{q}$. 4gggg. *Divide therefore* 2gg (*or Hq.*) (*by* kp *or*) C, *that is, make* C. H :: H. F. *therefore* C *in* F, *is* ∞ *to* 2gg (∞ Hq) *and* F *is the geometrical Quotient that riseth by this division. Therefore the square of* F, (*to wit*) Fq. *is* ∞ *to* $\frac{4gggg}{bb-dd}$ *to this adde* $\frac{1}{4}$bb *that is, make* rs (*rightangled at* r) ∞ $\frac{1}{2}$b, *and draw* ks (ks) *shall be the* $\sqrt{q}$. $\frac{4gggg}{bb-dd}$ + $\frac{1}{4}$bb, *from this take out* $\frac{1}{2}$d (∞ st) *the remainder* kt *is* ∞ *a, and if you adde* (sx) ∞ $\frac{1}{2}$d *to* kt. kx ∞ d + a. *Therefore from the three sides given* b. a. a + d *or* kn, kt, kx *form the triangle* n A k *agreable to the Scheme.*

PROBLEMA XXII.

Fig. 32. *Datis trianguli rectanguli summa hypotenusæ, & perpendiculi* (b) *& area* pp *invenire Basim.*

PROBLEM XXII.

In a rectangled triangle (b) the sum of the hypotenuse, and perpendicular are given, and pp the area, the bases is required.

Sit basis x

$\frac{2pp}{x}$ erit perpendiculum,

$b - \frac{2pp}{x}$ erit hypotenusa.

Ergo quadratum hypotenusæ bb

Let the Base be x,

the perpend. shall be $\frac{2PP}{x}$ &

b − $\frac{2PP}{x}$ *shall be the hypotenuse. And the square of the hypote-*

Fig. 32.

$bb - \frac{4bpp}{x} + \frac{4pppp}{xx}$ ∞ $xx + \frac{4pppp}{xx}$ & sublatis æquiponderantibus, & reducta æquatione. Erit xxx ∞ $x\,bb - 4bpp$. Sed utrum hæc æquatio cubica dummodo speciebus remanet obvoluta possit reduci, ad quadraticam difficulter judicatur. Datis speciebus applicabimus numeros ut in apposita figura. Sit b ∞ 8. & pp ∞ 6. Erit

xxx ∞ $64\,x - 192$, vel $xx - 64x + 192.$ ∞ $o\,o$.

Vel 1c. ∞ 64✓ − 192, variis modis solubilis. Nam si 1c. ∞ 64✓ − 192. Erit etiam 1q. ∞ 64✓ − $\frac{192}{1✓}$ Ergo pars aliquota $\frac{192}{1✓}$ subducta ex 64 relinquet numerum æqualem 1q. Partes aliquotæ 192 sunt eligatur ejusmodi quæ subducta ex 64 relinquet quadratum numeri collateralis.

Ex partibus aliquotis.

1 .	192
2 .	96
3 .	34
4 .	48
6 .	32
8 .	24
10 .	19.$\frac{1}{5}$

Experiamur. Subducatur 34 ex 64 relinquit 30, sed 30 non est quadratum 3. Subducamus

hypotenuse $bb - \frac{4bpp}{x} + \frac{4pppp}{xx}$ ∞ $xx + \frac{4pp}{xx}$ *let the terms of equal value be taken away, and then the equation reduced will be* xxx ∞ $x\,bb - 4bpp$. *Now whether this cubick equation whilst it thus remains hid under species can be reduced to a quadratick is hardly judged. Let us therefore apply numbers to the species, and let* b *be equal to* 8, *and* pp *equal to* 6.

xxx ∞ $64\,x - 192.$ *or* $xxx - 64\,xx + 192.$ ∞ $o\,o$.

Which equation may be resolved several ways. For if 1c. ∞ 64✓ − 192 *it followes that* 1q. ∞ 64 − $\frac{192}{1✓}$ *therefore, some aliquot part of* 192 *taken out of* 64 *shall leave a number equal to a square. The aliquot parts of* 192 *are as in the Table.*

1 .	192
2 .	96
3 .	34
4 .	48
6 .	32
8 .	24
10 .	19.$\frac{1}{5}$

Let us try, and first subduct 34 *out of* 64, *there remains* 30, *but* 30 *is not the square of* 3 *the*

camus ſecundo, 48 ex 64 relinquit 16, quadratum 4, numeri collateralis, ergo 1 q. ꝏ 16 & valor x 4.

the correſpondent number, therefore let us try the ſecond time, and ſubduct 48, there remaias 16, the ſquare of 4 the collateral number, therefore 1 q. ꝏ 16 and 1√ ꝏ 4.

Secundo. Quia antea inventa æquatio x x x − *b b* x + 4 *b p p*. ꝏ *o*. vel x x x − 64 x + 192. ꝏ *o o*. Quæratur binomium per quod æquatio dividatur abſque fracto quod invenietur x − 4 ſupponamus igitur x − 4. ꝏ *o o*, & partiatur æquatio hoc modo,

Secondly, Becauſe x x x − b b x + 4 b p p ꝏ *o o. or* x x x − 64 x + 192 ꝏ *o o. Seek a binome which will divide this equation without a fraction, which will be found* x − 4, *and the quotient will be as appeares.*

$$
\begin{array}{l}
x - 4.\ ꝏ\ o\,o)\ xxx - 64x + 192\ (xx + 4x - 48 \\
\qquad\qquad\quad \underline{xxx - 4xx} \\
\qquad\qquad\qquad + 4xx - 64x + 192 \\
\qquad\qquad\qquad \underline{+ 4xx - 16x} \\
\qquad\qquad\qquad\qquad - 48x + 192 \\
\qquad\qquad\qquad\qquad \underline{- 48x + 192} \\
\qquad\qquad\qquad\qquad\ \ o\,o + \ o\,o
\end{array}
$$

Ergo valor unius radicis eſt 4, ſed quia æquatio tres habet dimenſiones reſtant duæ adhuc aliæ deducendæ ex æquatione quadratica in quoto inventa ſuntque reliquæ duæ + √q. 52 − 2, & − √q. 52 − 2, altera affirmativa altera negativa, & ſic exprimantur, 2 ± 2. √ 13.

Therefore the value of one root is four. But becauſe the equation hath 3 roots by reoſon of its 3 dimenſions, there remains yet two to be deduced out of the quadraticks equation, and they are + √*q.* 52 − 2, *and* − √*q.* 52 − 2, *one affirmative the other negative, and may be thus expreſſed* − 2 ± 2. √ 13.

Fig. 33.

$$
\begin{array}{lll}
AB\ ꝏ\ 4 & \left\{\begin{array}{l} Ab\ ꝏ\ -2 + 2\cdot\sqrt{13} \\ Ac\ ꝏ\ + \dfrac{1 + \sqrt{13}}{2} \\ bc\ ꝏ\ + \dfrac{15 - \sqrt{13}}{2} \end{array}\right\} & \begin{array}{l} A\varepsilon\ ꝏ\ -2 - 2\sqrt{13} \\ A\gamma\ ꝏ\ + \dfrac{1 - \sqrt{13}}{2} \\ \varepsilon\gamma\ ꝏ\ + \dfrac{15 - \sqrt{13}}{2} \end{array} \\
Ac\ ꝏ\ 3 & & \\
BC\ ꝏ\ 5 & &
\end{array}
$$

Baſi

Fig. 33.

Basi igitur existente 4 triangulum erit A B C.

Basi existente $-2+2\sqrt{13}$ triangulum erit A *bc*.

Basis fuerit $-2-2\sqrt{13}$ triangulum erit A $\beta\gamma$.

In quibus omnibus area erit 6, summa hypotenusæ, & perpendiculi 8, sumptis quantitatibus antrorsum ab A ad B *b*, & ad C *c* pro affirmativis, retrorsum vero ad $\beta\gamma$ pro negativis.

Alii istiusmodi æquationes solvunt methodo (ut sic dicam) empyrico, seu tentativo. Hoc modo, sit 1c. ∞ 2√ + 4. Assumatur pro valore radicis: radix quilibet cubica exempli causa 2, ergo 1 c erit 8, & 8 debet esse equalis 2√ + 4, uti revera est, ergo 1 c ∞ 2√ + 4, hoc est 8, ∞ 4 + 4. Sit denuo 1c. ∞ 12 √ + 16.

Assumatur 2 vel 3 pro valore radicis unius invenientur minores justo nam cubus 3 est 27, ergo 27 debuit esse æqualis 36 + 16, *viz.* 52. Assumatur 4 pro radice, ergo 64 debet esse equalis 48 + 16 uti est: sin fuerit 1 c. ∞ 12√ + 20. 4 invenietur minor (5) justo major. Ergo valor erit inter 4 & 5, extrahatur radix cubica ex 48 + 20 (*viz.*) 68 adjectis cyphris, & habebis valorem radicis ut volueris precise.

The base therefore being 4 the triangle shall be A B C.

The base being $-2+2.\sqrt{13}$ *the triangle shall be A* b c.

The base being $-2-2.\sqrt{13}$ *the triangle shall be A* $\beta\gamma$.

In all which the area is 6, the sum of the hypotenuse and perpendicular 8, the quantities being taken forward from A to B b *and C* c *affirmative, but backward to* $\beta\gamma$ *negative.*

Others resolve these kinde of equations by an empyrical, and tentative way, as I may call it, not much unlike the first solution of this question. Suppose 1c ∞ 2√ + 4. *Assume for the value of* 1√ *the root of any cubical number whatsoever, as for example* 2. *then* 1c ∞ 2√ + 4. *shall be* 8 ∞ 4 + 4, *as in truth it is, therefore* 2 *is the value of one root.*

Again, Suppose 1c ∞ 12√ + 16. *Take* 2 *or* 3 *for the value of* 1√, *they will be found too little, for* 27 *the cube of* 3 *should be equal to* 36 + 16, viz. 52, *which it is not. Take* 4, *then* 64 *should be equal to* 48 + 16 *as indeed it is, therefore* 4 *is the value of* 1√, *but if* 1c *had been equal to* 12√ + 20, 4 *will be found too little, and* 5 *too big, therefore the value of* 1√ *is between these numbers. Therefore extract the cubick root of* 48 + 20, *viz.* 68 *adding*

Problema XXIII

Fig.34. *Datâ summa areæ paralellogrammi rectanguli, & diagonii, & data etiam differentia, vel summâ laterum, invenire singula.*

Problema est numerose solvendum alias enim dari non potest summa areæ & diagonii.

Data.

s ∞ 73. Summa areæ diagonii.

b ∞ 7. Differentia laterum.

Quæro latus minus.

Sit x ergo $x+b$ est latus majus, & $xx+xb$. est area rectanguli sed quadrata duorum laterum simul addita sunt æqualia quadrato hypotenusæ, per penul. 1 *Eucl.* Ergo $2xx+2xb+bb$ ∞ quadrato diagonii. Sed $xx+bx$ est area rectanguli, ergo quadratum diagonii æquatur duplo areæ rectanguli plus laterum differentia quadrata. Hoc est $2xx+2xb$. ∞ □ diagonii $-bb$. Eo igitur deventum ut ad solutionem hujus problematis nihil aliud requiratur quam ut dividamus (73) summam diagonii, & areæ in duas istiusmodi partes, ut quadratum unius minus 49 (bb) sit æquale duplo partis alterius.

Sit

cyphers, and you may have the root as precisely as you desire.

Problem XXIII.

The sum of the area of a rectangle parallelogram, and the diagonium being given, as also the difference, or sum of the sides being given, to find the rest.

This Probleme must be resolved in numbers, otherwise the sum of the diagonal and area cannot be given.

Given. s ∞ 73. *sum of the area and diagonal.*

b ∞ 7. *The differ. of the sides.*

I seek the lesser side.

Let it be x. *therefore* $x+b$ *is the greater, and* $xx+xb$ *is the area of the parallelogram. But the aggregate of the squares of both the sides are equal to the square of the diagonal, by the* 47 1 Eucl. *Therefore* $2xx+2xb+bb$. ∞ *to the* □ *of the diagonum, but* $xx+bx$ *is the area of the parallelogram Therefore the square of the diagonal is equal to double the area of the parallelogram & the square of the difference of the sides. That is* $2xx+2xb$. ∞ □ *diag.* $-bb$. *Therfore for the solution of this question there is no more required then to divide* (73) *the sum of the area and diagonal into* 2 *such parts, that the square of one of them, lessened by* 49 ∞ bb *shall be equal to double the other part.*

Let

Sit jam x. pars una(*sc.*)diagonium erit $73 - x$ pars altera, *viz.* area, & $xx - 49. \infty 146 - 2x$, vel $xx \infty 195 - 2x$, & $\sqrt{q}.196 - 1 \infty x \sqrt{q}.196$ est 14, tolle 1 erit 13 diagonium : & $73 - 13$. *viz.* 60 erit area. Hinc oritur novum problema.

Quære duos numeros differentes per 7, qui invicem multiplicati producant 60. Sit primus & minor numerus y, major erit $y + 7$ & $yy + 7y. \infty 60$. Ergo $\sqrt{q}.\frac{49}{4} + 60$ hoc est $\frac{289}{4}$ *viz.* $\frac{17}{2} - \frac{7}{2}$, hoc est $\frac{10}{2} \infty 5$, est minor numer. Ergo major erit.

Majus latus parallelogrammi erit	12
Minus latus	5
Diagonium	13
Area	60

Let x *be one part, to wit, the diagonal* $73 - x$ *shal be the area, and* $xx - 49$ *shall be equal to* $146 - 2x$. *or* $xx \infty 195 - 2x$. *Therefore* $\sqrt{q}.\ 196. - 1$, *to wit* 13, *shall be the diagonal sought. and* $73 - 13$ *to wit* 60, *shall be the area. From hence arises a new Probleme.* Fig. 34.

Prob. *What two numbers are they whose difference is* 7, *and the product of them* 60, *which are easily found to be* 5, *the lesser & 12 the greater, so that*

The greater side of the parallelogram is	12
The lesser	5
The area	60
The diagonal	13

Sin aliter rem tentaveris in magis operosam divenies æquationem. Nam sit 73, summa areæ, & diagonii, & laterum differentia sit 7, erit $xx + 7x$ area, ergo $73 - xx - 7x$ erit diagonium, cujus quadratum erit $x^4 + 14x^3 - 97x^2 - 1022x + 5329 \infty 2x^2 + 14x + 49$, vel post debitam terminorum transpositionem, & reductionem, erit

$x^4 \infty - 14x^3 + 99xx + 1036x - 5280$, per communem *Algebræ* regulam, radix x^4 invenire non potest, invenietur tamen methodo in problemate precedenti indicata.

Supponamus $x \infty 3$. x^4 erit $\infty 81$. Invenietur minor justo.

Supponamus secundo $x \infty 5$. x^4 erit 625.

$99xx. \infty$	2475
$1036x \infty$	5180
summa	7655
$14x^3 \infty$	1750
5280∞	5280
summa	7030
Tolle ex	7655
Restant	625

Ergo recte divinavimus.

If you go about to solve this Probleme otherwise, you will at last come to this Equation.

$x^4 \infty - 14x^3 + 99x^2 + 1036x - 5280$, *whose root will be found by the method propounded in the preceding Probleme.*

$\infty 625$.

Sit

Fig. 43. *Sit jam data summa laterum* (s ꝏ 17) *summa areæ*, & *hypotenusæ* 73. *Quærantur reliqua.*

Sit latus minus x, latus majus erit $s-x$, area erit $sx-xx$, $ss-2sx+2xx$ quadratum diagonii, vel ss. ꝏ $2sx-2xx$. hoc est duplo areæ parallelogrammi. Ergo quadratum summæ (73q.) minus duplo areæ parallelogrammi est quadratum diagonii.

Hinc oritur novum Problema. Divide 73 in duas istiusmodi partes ut duplum unius sit æquale quadrato alterius (289. ꝏ ss) & omnia invenientur ut in problemate præcedenti.

Let (s ꝏ 17) the sum of the sides be given, as also 73 the sum of the area and diagonum. The rest are sought.

Let the lesser side be x, *the greater shall be* $s-x$. *The area shall be* $sx-xx$. $ss-2sx+2xx$ *shall be the square of the diagonium, or* ss ꝏ $2sx+2xx$, *that is double the area of the parallelogram. Therefore the square of the sum* (73q.) *lessened by double the area shall be the square of the diagonium.*

Hence ariseth a new Probleme Divide 73 *into two such parts that the double of one may be equal to the square of the other* (289 ꝏ ss) *and every thing will again be found as in the precedent Probleme.*

FINIS.

Fig. 18

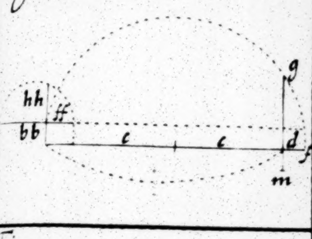


Fig. 19

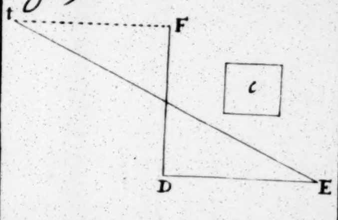


Fig. 20

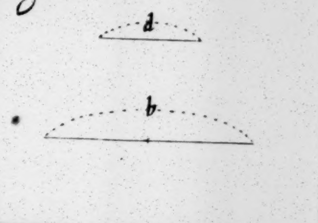


Fig. 21

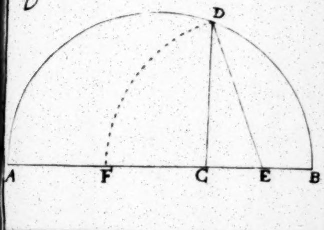


Fig. 22

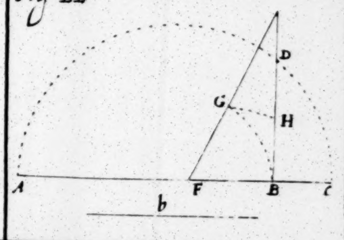


Fig. 23

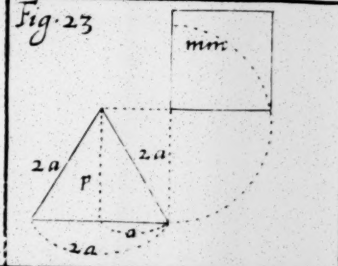


Fig. 24

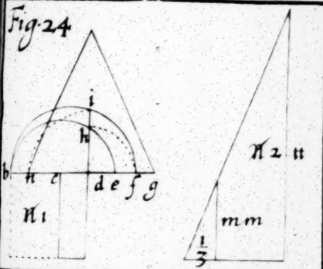


Fig. 25

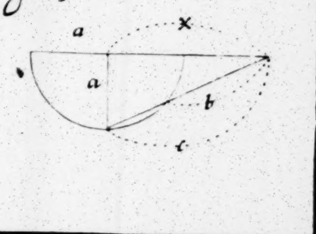


Fig. 26

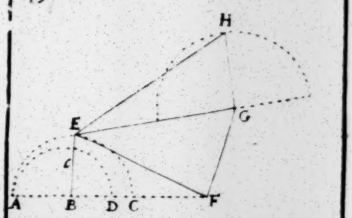


Fig. 27

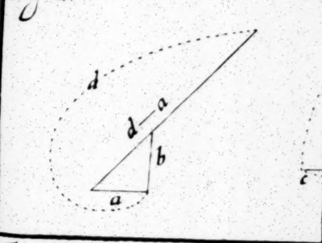


Fig. 28

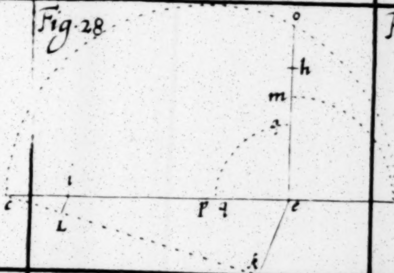


Fig. 29

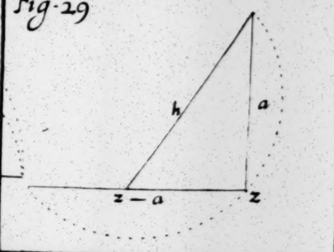


Fig. 30

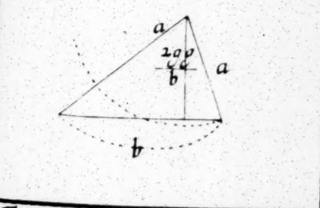


Fig. 31

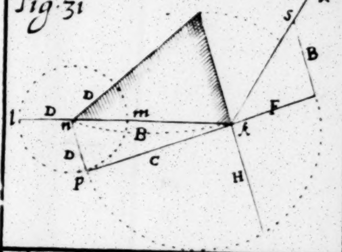


Fig. 32

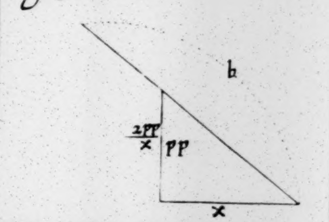


Fig 33

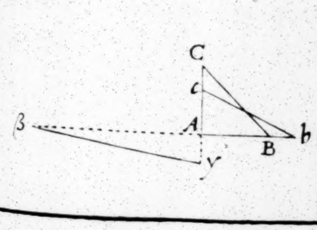
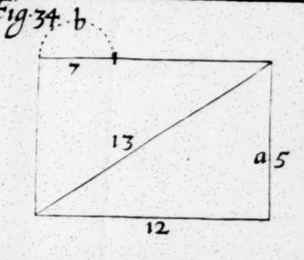


Fig. 34



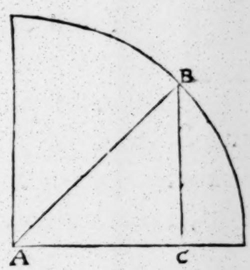
Place this at the end of Problematum quorundam Mathematicorum.

PROBLEMATA
Quædam ſuccincta condendi Canones Sinuum, Tangentium, & Secantium.

PROBLEMA I.

Dato Sinu arcûs, Sinum complementi reperiri.

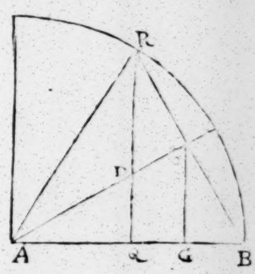
DAto B C invenire A C. Quoniam Triangulum ACB eſt rectangulum (per ſinus definitionem) & latera A C, B C, æquè poſſunt hypotenuſæ, id eſt, radio A B: ſi igitur quadratum Sinus B C ſubtrahatur de Quadrato radij A B, relinquitur quadratum A B, cujus latus eſt recta A C, ſinus quæſitus.



PROBLEMA II.

Dato Sinu arcûs, unà cum ſinu complementi, ſinum arcûs dimidii reperire.

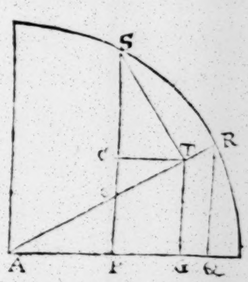
DAtis R Q, A Q invenire B O vel R O. Ut A *B* ad *B* O, ita *B* O ad *B* G. Erit ergo *B* O latus Quadratum plani ex A *B* radio & *B* G ſemiſinu verſo dato. Datur enim Q *B* ſinus verſus arcûs *B* R, quia A Q ſinus complementi, & A *B* radius dantur ex hypotheſi.



PROBLEMA III.

Datis ſinibus duorum arcuum, & ſinibus complementorum, ſinum ſummæ reperire.

DAtis R Q, Q A, & S T, T A, quæratur S P. Ut A R ad R Q, ita A T ad T G, ſive C P. Ut A R ad A Q, ita S T ad S C. S T & C P ſimul, faciunt S *P* ſinum ſummæ duorum arcuum.



PROBLEMA IV.

Eisdem datis, sinum differentiæ reperire.

DAtis R Q, Q A, & *S P*, *P* A, quæratur *S* T. Ut A Q ad Q R, ita A *P* ad *P* O, unde innotescet O *S*. Ut A R ad A Q, ita O *S* ad *S* T.

His adnectantur Theoremata.

Theorema I. *Sinus minimi sunt in ratione suorum arcuum ferè.*

HOc Theorema verum esse infrà ostendetur in bisectionibus continuis. Arcus autem minimi sunt unius circiter scrupuli primi, vel infrà. *S*unt ferè in eadem ratione qua & sinus sui, quia inter se ferè contigui ejusdemque adeò quantitatis propemodum, ad scrupulositatem satis profundam, non autem omnimodam.

Theorema II. *Si eadem linea secetur in partes numero inæquales, numerus partium primæ sectionis ad numerum secundæ, est (reciprocè) prout pars una sectionis secundæ, ad unam partem sectionis primæ.*

SEcetur eadem linea, primò in 4, deinde in 3 partes: Erit igitur Ut 4 ad 3, ita $\frac{1}{3}$ pars ad $\frac{1}{4}$ reciprocè. Ratio est quia 3 in $\frac{1}{3}$ facit 1, item 4 in $\frac{1}{4}$ facit 1. Quandoquidem verò facti sunt æquales, erunt factores reciprocè proportionales, per 6 *Eucl.*

Structura Canonis Sinuum.

TOtius quadrantis sinus, Radius dicitur; est enim semidiameter circuli. *S*tatuatur autem in Canone Radius 100000 partium, vel etiam 100000.00, pro calculi necessitate. Ad structuram autem Canonis commodiùs assumitur partium 100000.00000, ita enim errores qui in dextimas figuras subrepunt deleri tutò possunt absque Canonis præjudicio.

Bisecetur deinde quadrans, & bisegmenti exquiratur sinus, per *Probl.* 2. ejusque cosinus per P*robl.* 1. Hoc rursus bisegmentum

segmentum bisecetur, & secundi bisegmenti investigetur sinus per *Probl.* 2. cosinus etiam per *Probl.* 1. Porrò & secundum hoc bisegmentum bisecetur, & investigentur ejusdem sinus & cosinus, per *Probl.* 2 & 1. Deinde verò & tertium bisegmentum bisecetur &c. continueturque bisectio tredecies, usque dum inventus sit sinus $\frac{1}{8192}$ partis totius quadrantis, prout hic in Tabella apponitur. Jam verò ad arcus minimos diventum est, ubi Theorematis primi veritas illustratur; Nam, Ut arcus quadrantis $\frac{1}{4096}$ est duplus ad arcum $\frac{1}{8192}$, ita & illius sinus ferè ad sinum hujus.

Quadrantis sinus	100000.00000
$\frac{1}{2}$ quadrant. sinus	70710.67811 +
$\frac{1}{4}$ quadrant. sinus	38268.34323 +
$\frac{1}{8}$ partis quadr. sin.	19509.03220 +
$\frac{1}{16}$ partis quadr. sin.	9801.71403 +
$\frac{1}{32}$	
$\frac{1}{64}$	
$\frac{1}{128}$	
$\frac{1}{256}$	
$\frac{1}{512}$	
$\frac{1}{1024}$	
$\frac{1}{2048}$	
$\frac{1}{4096}$	
$\frac{1}{8192}$	

Post sinum hunc minimum sic inventum, inveniendus etiam est sinus unius scrupuli primi, id est, $\frac{1}{5400}$ partis de toto quadrante; vel unius centesimæ partis gradus, id est $\frac{1}{9000}$ partis totius quadrantis. Juxta igitur Theorema 2; Ut $\frac{5400}{9000}$ ad 8192, ita quantitas 1 partis hujus divisionis ad quantitatem 1 partis divisionis illius, & per Theorema 1, ita sinus $\frac{1}{8192}$ partis quam habes in Tabella ad sinum $\left\{\begin{matrix}\frac{1}{60} \\ \frac{1}{100}\end{matrix}\right.$ partis unius gradus.

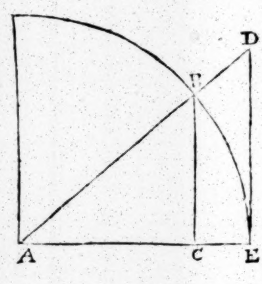
Sinu

Sinu igitur 1 minuti, vel 1 centeſimæ partis ita formato, per *Probl.* 1. erue ſinum complementi, arcus ſcilicet 89 gr. {59/60 {99/100 Deinde, per *Probl.* 3. exquire ſinum 2 min. ejuſque coſinum per *Problem.* 1. Et ex his invenies ſinum ſummæ 2 m. & 1 m. id eſt 3 min. per *Probl.* 3. ejuſque rurſus coſinum per *Probl.* 1. Ex ſinu autem & coſinu 2 m. ſive ex ſinibus & coſinibus. 3m. & 1 m. inveſtigabis ſinum 4m. per *Probl.* 3. & ſinum complementi per *Probl.* 1. Item ex ſinibus & coſinibus 2 m. & 3 m. vel 4 m. & 1 m. invenies ſinum & coſinum 5 m. per *Probl.* 3 & 1 &c. uſque ad {60′/100 vel 1 gradum. Ex ſinu etiam gradus unius poteris eiſdem mediis reperire omnes ſinus 90 graduum integrorum: & ex priùs inventis ſinibus & coſinibus minutorum 60′ ſingulorum, facile erit per *Probl.* 3. adhibito etiam *Probl.* 4. quando è re fuerit, eruere ſingulorum omnium minutorum interſperſorum ſinus ſingulos.

Tangentium & *Secantium* deductio è Tabulis *Sinuum.*

Tangentes formantur ſic.

Ut A C *coſinus, ad* C B *ſinum; ita* AE *radius, ad* E D *Tangentem.*



Secantes autem ſic.

Ut A C *coſinus, ad* AB *radium; ita* AE *radius, ad* A D *Secantem.*

Hoc modo integri Canones Tangentium & Secantium è ſinuum Canone eliciuntur.

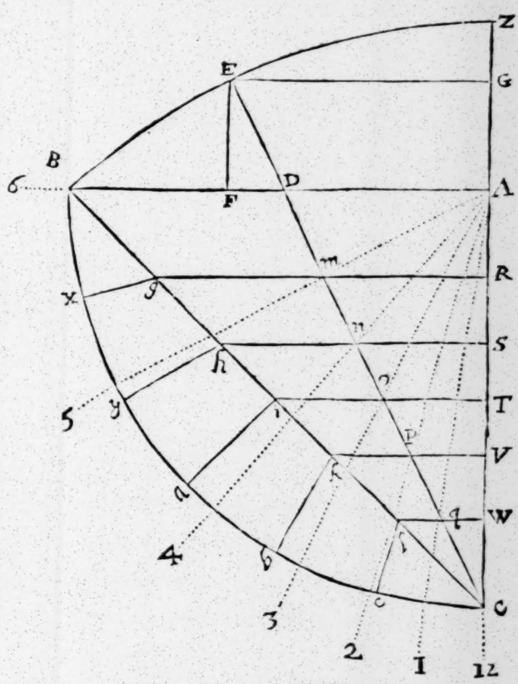
Compendia calculi prætermittimus omnia, Canones enim de novo condere non aggredimur; quandoquidem præſtantiſſimorum Artificum pertinaci ſtudio & labore hoc faſce liberamur. Noſtro ſufficit inſtituto ſi Syntaxeos ratio qualiſcunque tantummodo intelligatur, & veritas numerorum in Canonem ingeſtorum: quod Propoſitiones ſupra poſitæ abundè comprobant.

F I N I S.

Demonstratio Quadrantis
HOROMETRICI.

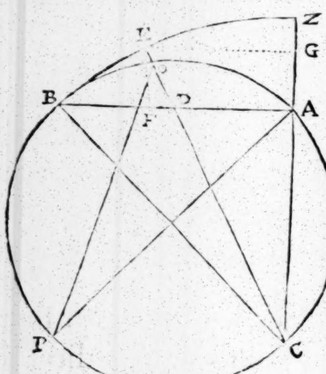
Quandoquidem visum est Viro Erudito D. Francisco à Schooten, Leydensi, *in Academia* Lugduno-Batava *Matheseos Professori, quadrantem Horometricum ab Authore nostro ante plures annos excogitatum; anno autem* 1638 Anglico *sermone impressum non solum laudare, sed praxeos veritatem ingeniosa demonstratione munire Sect.* Miscellan. *pag.* 510. *Placuit nobis Authoris ipsius demonstrationem qualem inter adversaria reperimus hic etiam subnectere.*

Sit radius AB vel AC: BC Horarum linea (in quadrante) artificialitèr divisa per filum *c* A, *b* A, *a* A, *y* A, *x* A, in punctis *l k i h g*: & ducantur *l* W, *k* V, *i* T, *h* S, *g* R, parallelæ ad BA. Sit porrò AD sinus 30 *gr.* respectu radii A*B*, & agatur recta *C* DE, quæ quidem dividet unamquamque rectarum parallelarum *g* R, *h* S, &c. in partes similes *B* D, D A; adeoque, Ut A*B* radius ad A D sinum 30 *gr.* ita R *g*, ad R *m*, & ita S *h*, ad S *n*, &c. Erunt ergo, Ut W *l* radius, ad W *q* sinum 30 *grad.* ita W *l* tangens 15 *gr.* ad W *q* tangentem anguli 1 A 12. & sic in horis reliquis.



Porrò autem in nostro Quadrante recta *C* D ponitur semper sub eadem longitudine cum *C B*, perindè ac si radio *C B* describeretur semiquadrans *B* E Z, & vice C D usurpatur

C E, pro C A etiam ponitur C G. Utcunque tamen triangulo E G C, & D A C sunt similia, & quoniam E C divisa sit in partes easdem cum partibus *B* C, dividetur etiam in partes similes iis quas continet recta D C; atque adeò idem opus absolvet. Linea igitur nostra latitudinum tota est A B, partes verò non sunt sinus A D, &c. sed G E, &c. vel *A* F, &c. designatæ per rectam C *D* protensam in E circumferentiam, ut C E sit æqualis *C* B. Inquiruntur autem hoc modo. Summæ quadratorum radii *C A*, & sinus *A D*, radix erit *C D*; Ut verò *C D* ad *D A*, ita *C E* = *C* B ad rectam E G, quæ inscribenda est lineæ latitudinum *A* B ad F; & *A* F erit pro latitudine 30 *gr*. Exempli gratiâ. Quadratum *A D* est 25000000000000, quadratum *AC* est — 100000000000000, summa quadratorum 125000000000000, cujus radix est *C D* recta — 11180340. At verò, Ut *C D* 11180340, ad *D A* 5000000; ita *C* B vel *C E* 14142136, ad E G vel A F 6324555. Tanta igitur est recta A F respondens 30 *grad.* in linea latitudinum. Et sic de partibus reliquis. Vel, Ut *C A* radius 100000, ad *A D* sinum 30 *gr*. 50000: ita *C A* radius, ad *A D* tangentem anguli *A C D* 26 *gr*. 33′ 54″. Hoc est, sinus *A D* ingestus in canonem Tangentium, dat arcum 26 *gr*. 33′ 54″, cujus sinus est 4472128; Atque posito radio *C* B = *C E* = *C Z*, *A* B est sinus 45 *gr*. 7071068, ideò rursus augendus est sinus 4472128, hâc ratione; Ut sinus 45 *grad.* 7071068 ad radium 100000 (vel, Ut rad. 10000000 ad secantem 45 *gr*. 14142136) ita 4472128 ad 6324544, quæ est longitudo rectæ E G, vel *A F*, ferè ut suprà. Superior autem operatio produxit paulò accurationem. Hæc autem inquisitio usui abundè satisfaciet.



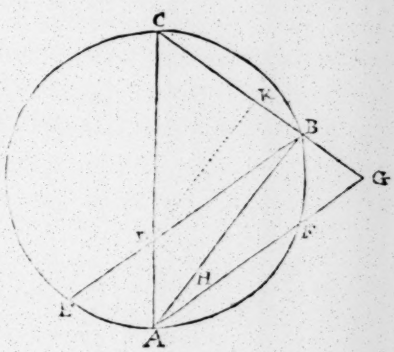
Hoc præterea non omittendum. *C* B est linea Horarum quadrantis, & *A* B est linea latitudinum. Duo igitur, si circulus in posteriori parte describatur super *C* B, æqualis nempè diametri cum linea horarum, chordæ quadrantis 90 sinum in circulo, erunt eædem cum partibus 90, lineæ latitudinum. Nam (exempli gratiâ) ad *A* B radium, sit

sit A D sinus 30 *grad.* C D O E secabit circulum in O; peripheriam Z B, in E; & efficiet tum E G (vel F A) 30 *gr.* in linea latit. (quod suprà probatur) tam A O 30 *gr.* in quadrante circuli A O B. At chorda A O, æquatur rectæ E G. Nam <, O P A, & O C A sunt equales, quod sunt in peripheria ad P & C, & insistunt eidem arcui O A. *Prætereà,* P *A*, & *C* B vel *C* E sunt æquales; & *P* O *A*, *C* G E, sunt < recti. Ergo (cum *P* O *A*.*C* G E, sunt similia, vel æquiangula, latera homologa) E G, *A* O sunt æqualia. Quod probandum erat.

Demonstratio faciei posterioris Horometrici Quadrantis, adeoque Instrumenti totius Circularis.

Theorema I. *Si à diameter, diametrum circuli secet, erunt segmenta diametri proportionalia tangentibus arcuum oppositorum diametri segmentis conterminorum.*

SIt à diameter B E secans diametrum C A in D, dico primò, Ut segmentum C D ad D A, ita tang. arcus C B ad tang. E A. Notandum autem totum circulum hic dividi tantummodò in 180 *gr.* semicirculum in 90. quia de arcubus hic agitur prout angulos in peripheria obeunt, quorum sunt tantum subdupli.

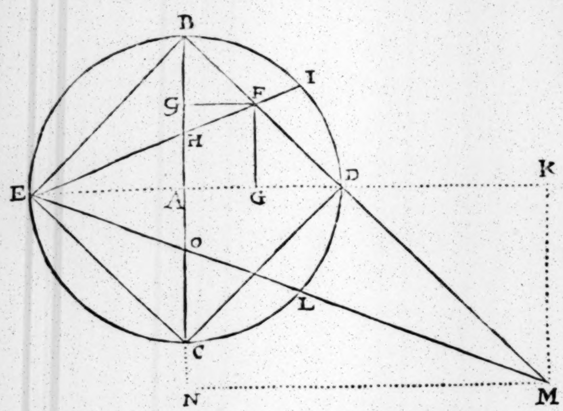


Demonstrat. Fiat enim A G parallela ad adiametrum B E, & ducantur A B, C B G. Primum igitur quia C B A est rectus (in semicirculo quippe) erunt C B & B G tangentes angulorum C *A B*, *B A* G respectu radii *A B*, id est arcuum C *B*, & *B* F = E *A* quia uterque B F, E *A* includitur inter parallelas *B* E, G *A*. Deinde quia B E & G *A* sunt parallelæ, erit Ut C D segmentum, ad D *A* segmentum, ita recta *B* C tang. arcus B C, ad rectam B G tangentem arcus B F = E *A*. Dico secundò: Ut C D ad *D A*, ita tangens arcus C E, ad tangentem arcus oppositi *A B*, quod sic facile evincitur. Quia C B & B *A*, item C E, E *A*, sunt sibi invicem complementa, quorum tangentes sunt reciprocè proportionalia. Quare tangens C B ad

ad tang. E *A* est in eadem ratione qua cotang. E *A* (id est tang. C *E*) ad cotang. C B (id est tang. B A,) ergo & tangentes C *E*, B A, sunt ut C D, D A.

Aliò modo sic evinco, Fiat D H perpendicularis ad B A, ac proinde parallela ad C B. Si DH sit radius, erunt HB, H A, tangentes angulorum *B* D H, H D A, id est arcuum C E, *B A*. C D. D A : : B H tangent. *B* D H = C B D = C *E*. H *A* tang. H D *A* = B C A = B A. Nam *BD*H est complementum D *B* H vel arcus E *A*, cujus complementum est etiam arcus C E ; item D A H vel C B arcus est complementum ADH vel B A arcus, ergo A D *H* & B A æquantur. Et quia C B & B H sunt parallelæ, ergo C *D*. D A : : B H tang. C E. H A tang. B A. Rursus per dimissionem perpendiculi D K. Ut C *D*. *D* A : : C K tang. C D K = C A B = C *B*. K B tang. K D *B* = D *B* H = E A.

Theorema II. *Si inter duas parallelas duæ rectæ ducantur se mutuò secantes , segmenta unius erunt proportionalia segmentis alterius, si similiter utrobique capiantur.*



INter parallelas *BD*, E C, ducantur *B*C, *E* F, se mutuò secantes in H, dico segmenta C H, H *B*, esse proportionalia segmentis E H, H F. Ratio est. Quia triangula *E* H C, F H *B* ; sunt æquiangula , propter æquales ad verticem H, & alternos æquales H C E, H *B* F ; item H E C, H F *B*, alternos nempè inter parallelas B D, *E* C. Quapropter Ut C *H*, ad H *E*; ita H B, ad H F: & alternatim, Ut C H, ad H B; ita H E, ad *H* F.

Theorema II. *Si quadrati, diagonio intersecti, latus unum infinitè continuetur ; & ab angulo utrique apposito, in continuatum recta ducatur, secans & continuatum & diagonium; erunt segmenta diagonii ut radius ad perpendiculum inter segmentum continuati, & diagonum: vel ut quadrati latus ad segmentum continuati diagonio conterminum.*

Demon-

Demonſt. SIt quadrati E *B* D C diagonium *B* C (prolongatum ſi opus fuerit) & latus *B* D infinite continuatum, & ab E angulo utrique oppoſito ducatur recta $\left\{\begin{matrix} E\ F \\ E\ M \end{matrix}\right.$ ſecans continuatum in $\left\{\begin{matrix} F \\ M \end{matrix}\right.$ diagonium autem in $\left\{\begin{matrix} H \\ O \end{matrix}\right.$ dico ſegmenta diagonii $\left\{\begin{matrix} C\ H \\ C\ O \end{matrix}\right.$ & $\left\{\begin{matrix} H\ B \\ O\ B \end{matrix}\right.$ ita eſſe inter ſe, ut eſt E A radius quadrati, ad $\left\{\begin{matrix} S\ F = A\ G \\ N\ M = A\ K \end{matrix}\right.$ perpendiculum inter ſegmentum ad $\left\{\begin{matrix} F \\ M \end{matrix}\right.$ & diagonium *B C*. Nam (per 2 præcedens) $\left\{\begin{matrix} C\ H \\ C\ O \end{matrix}\right.$ eſt ad $\left\{\begin{matrix} H\ B \\ O\ B \end{matrix}\right.$ ut $\left\{\begin{matrix} E\ H \\ E\ O \end{matrix}\right.$ ad $\left\{\begin{matrix} H\ F \\ O\ M \end{matrix}\right.$ vel ut E A ad $\left\{\begin{matrix} A\ G = S\ F \\ A\ K = N\ M \end{matrix}\right.$ Quod erat probandum.

Pars poſterior ſic cogitur. Quia, Ut *E* A, ad $\left\{\begin{matrix} A\ G \\ A\ K, \end{matrix}\right.$ vel, Ut D A ad $\left\{\begin{matrix} A\ G \\ A\ K = N\ M \end{matrix}\right.$ ita D B latus quadrati, ad $\left\{\begin{matrix} B\ F \\ B\ M. \end{matrix}\right.$

Corollar. 1. *Hinc* ſequitur. Si latus continuatum dividatur in partes quaſcunque (ſive æquales, ſive radices quadratas ſive ſolidas, tangentes, ſinuſve rectos, vel verſos) erit diagonium etiam in partes ejuſdem nominis ſectum atque tali modo, ut ſegmenta ſe ſemper habebunt ut latus quadrati ad partes continuato lateri inſcriptas, ſive ut radius quadrati ad longitudinem perpendiculi cujuſque prædicti, ſi ſegmenta ſumantur prout inter ſe reſpondeant. Cauſa manifeſta eſt è ſuperioribus.

¶ Notetur etiam (ſi cui bono) *H B* eſſe medium proportionale inter *H* F & *H* I. Nam ut *C H* ad *H E*, ita *H* I ad *H B*, per 3 *Eucl.* &, Ut *C H* ad *H* E; ita *H B* ad *H* F, ergo Ut *H* I ad *H B*; ita *H B* ad *H* F.

Corollar. 2. *Hinc* etiam. Si quadrato circumſcribatur circulus, partes cujuſcunque nominis projiciuntur à latere quadrati continuato in peripheriam; atque eo etiam modo, ita ut tangens quadrantis ſive radius ad tangentes partium

inscriptarum eandem semper servabit rationem quam tenet latus quadrati ad segmenta continuati lateris, sive radius ad partes inscriptas.

Demonstr. SIt enim F pars in latere quadrati, inscripta in circuli punctum *I*. Erit (per 1) Ut C *H* ad *H B*, hoc est (per 2) Ut radius E B ad partem inscriptam B F, vel sicut radius D A ad rectam A G; ita tangens quadrantis *E C*, hoc est radius rursum, ad tangentem arcus *B I*, cujus tangens erit ideo æqualis A G rectæ.

Modus inserendi. Ex his apparet modus inserendi partes omnis generis, *viz.* Sinuum, Tangentium, partium æqualium, radium quadratum, cubicarum, sinuum versorum, &c.

In Peripheria sic agendum est :

Omnes numeri cujuscunque generis ut B F, B D, B M, &c. ingesti in canonem tangentium dabunt arcus B I, B D, B L, &c. æquales pro tangentibus, inæquales pro numeris quibuscunque reliquis.

In diametro, sic :

Ut E B radius, & B F simul additi, ad B F partem radio additam; vel, Ut E G composita ex E A radio, & A G parte quacunque ad eandem rectam A G; ita diameter B C, ad segmentum B H.

Patet hinc, Partes non inseri ultrà quadrantem in circulo, ultrà radium in diametro, si modò intrà radium sive 100000 se contineant, quales sunt numerorum seu partium æqualium, Sinuum rectorum, Semisinuum versorum, Superficierum, Solidorum &c. At vero partes rectarum infinitarum quales sunt tangentes & secantes per totum omninò semicirculum, totamque adeò diametrum diffundi: partes rursum ad duplum radii extensas, occupare $\frac{2}{3}$ diametri, peripheriæ autem semicircularis paulò plus duabus tertiis. Hinc rursum, Quia est Ut tangens ad radium, ita radius ad cotangentem, perinde erit si dicas; Ut B H tangens ad H C radium, vel, Ut B H radius ad H C cotangentem; Nam segmenta diametri B H & H C representant tangentes angulorum B E I, I E C, qui se mutuò complent.

Theor. 4. *Vbicunque punctum suscipiatur in diametro, segmenta sunt ut radius ad partes numero affixo denotatas; Vel contra, Vt partes ad radium, hoc est, Vt radius ad partes complementarias (ut ita dicam.)*

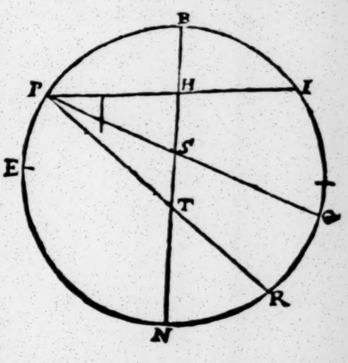
Ergo perinde est sive dicam, Ut radius ad partes, sive, Ut partes complementariæ ad radium. Analogicè dictum, juxta Theorema, Ut radius ad tangentem, sive Ut tangens ad radium. Hinc autem operandi methodus elucescet.

Modus operandi in istiusmodi lineis potest esse varius, est tamen unicus nobis & simplex. Fundatur autem in proportione Theor. 1. Quia segmenta diametri sunt ut ejus contermini arcus oppositi. *Modus operandi.*

PROPOSITIO.

Filo suprà planiciem circuli tenso, erunt radius, duo arcus, & segmentum diametri ab unà quâcunque parte, quatuor proportionales.

Demonstr. HÆc Propositio est omnium operationum basis. Dico, Ut radius ad P *B*, ita B *I* ad B *H*. Vel, Ut radius ad P N, ita *N I* ad *N H*. Quia enim est, Ut tangens *P N* ad radium *E N*, ita tangens *I B*, ad segmentum *B H*, per Theor. 1. Et ut tangens *P N* ad radium E N, ita radius E N ad tangentem complementi P B; per Compend. Trigonometr. Erit ergo, Ut radius ad *P B*, ita *B I* ad *B H*. Eademque ratione, Ut radius ad *P B*, ita *B* Q ad B S, & ita B R ad B T. Quapropter etiam, filo ab ima parte ad punctum aliquod peripheriæ fixo, ab altera parte per peripheriam oppositam diametrumque moto.



Erunt arcus omnes cum segmentis diametri, proportionalis.

Demonstr. NAm sunt omnes, Ut radius ad *P B*, per præcedens. Ergo, Ut *B I* ad *B H*, ita B Q ad B S, & ita B R ad B T, &c. Vel contrà, Ut *P* B ad radium, ita *B H* ad *B I*, ita B S ad B Q, ita B T ad B R, &c.

Corollar.

Corollar. E tribus igitur terminis datis, ſilum per duos priores (quorum alter in peripheria alter autem in diametro numerandus eſt) debito peripheriæ loco ſigendum eſt; hinc autem à parte alterâ ſi moveatur in terminum tertium ſuper eâdem circuli parte cum primo numeratum, exhibebit quartum in eadem circuli parte qua ſuſceptus erat terminus ſecundus. Demonſtratio hujus facilè reſultabit ex ſuperioribus.

Poterit etiam operatio inſtitui juxta mentem Theorematis primi: Eam autem hic repetere non erit operæ-pretium.

Notandum etiam eſt: Quamvis propriè latet myſterium operationis in Tangentibus, diffunditur tamen in partes aliarum denominationum, & puta Sinuum, Superficierum, &c. Quod quidem fit applicationo harum partium ad Tangentes quæ longitudines earum emetiuntur: quemadmodum in ſectore, operationes propriè pertinent ad lineas Æqualium partium, exindè verò derivantur in lineas ſuperficierum & ſolidarum, quia harum ſcalarum partes ex æqualibus partibus ſunt excerptæ, adeoque ſub eodem operis modo cadunt.

Quæ hìc obſcurè & ἀμεθόδως tradita ſunt, ſpero ſecundâ ſub recognitione planiùs & limatiùs proditura. Nam quæ exaſciata ſolummodò hic ſunt, erunt olim meditationibus maturioribus dedolata.

F I N I S.

EPITOME ARISTARCHI SAMII

De Magnitudinibus, & Distantiis trium Corporum,

SOLIS, LUNÆ, & TERRÆ.

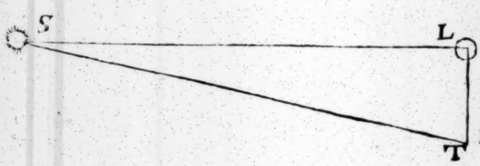
POSITIONES I.

Lunam à Sole lumen accipere. *II.* Terram puncti ac Centri, habere rationem ad Sphæram Lunæ. *III.* Cum Lunæ apparet nobis dimidiata vergere in visum nostrum Circulum maximum qui Lunæ opacum, & splendidum determinat. *IV.* Eodem dichotomiæ momento Lunam à Sole distare minus quadrante, parte ejusdem trigesima, vel 3 gradibus distat, ergo 87 *gr.* circiter. *V.* Umbræ latitudinem esse duarum Lunarum (id est 4 *gr.* per positionem sequentem.) *VI.* Lunam subtendere $\frac{1}{15}$ signi, id est 2 *gr.* De hâc positione vide *Archimedem*, in libro de Numero Arenæ, ubi diameter Solis (ex *Aristarcho*) decernitur esse $\frac{1}{720}$ pars circuli, id est $\frac{1}{60}$ signi, & sic *Aristarchum* allegat. *Keplerus* Epitom. pag. 476.

Pappus libro 6 Mathematicar. Collectionum pag. 136. ait, poſitiones 1, 3, 4 ferè, cum *Hipparchi*, & *Ptolomei* poſitionibus conſentire reliquas autem 2, 5, & 6 diſcrepare.

PROPOSITIO. VII.

Diſtantia Solis à Terra eſt $19\frac{11}{100}$ *pla. diſtantiæ Lunæ à Terra.*



ANgulus enim S T L eſt 87 *gr.* per *Propoſ.* 4. & L rectus, ergo ſi T L ſit radius, erit T S ſecans *gr.* 87 (ſc.) $19\frac{11}{100}$ ferè.

PROPOSITIO. VIII.

APparentes diametri Solis & Lunæ ſunt æquales, quia Sol totus in Eclipſi centrali deficit, at ſine morâ etiam quod obſervationes confirmant.

PROPOSITIO. IX.

Solis igitur diameter vera eſt $19.\frac{11}{100}$ *pla. diametri Lunæ.*

DIametri enim ſub eodem angulo apparentes erunt ut earundem diſtantiæ. Ut enim *u i* 1 ad *u a* $19\frac{11}{100}$: ita *i o* ad *a e*.

PROPOSITIO. X.

SOl ad Lunam eſt ferè, Ut 6979 ad 1. *S*unt enim, Ut cubi $19\frac{11}{100}$ & *i*, id eſt, Ut 6979 ad 1.

PROPOSITIO. XI.

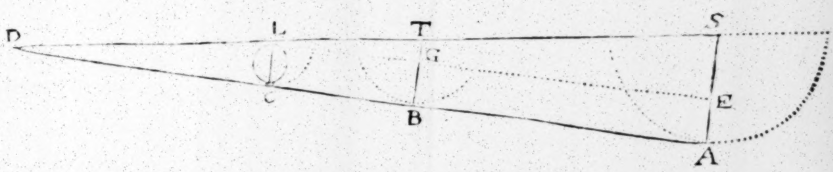
Diameter Lunæ est $\frac{7}{200}$ ma distantiæ Lunæ à Terrâ circiter.

DIameter enim apparens Lunæ est 2 *gr.* per *Posit.* 6. at subtensa 2 *gr.* est ad radium, Ut 35 ad 100 ferè, hoc est, ut 7 ad 200.

PRROPOSITIO. XV.

Solis diameter ad diametrum Terræ est, ut 382 *ad* 57.

QUoniam enim diameter Lunæ, L C æquat. dimidium diametri umbræ (per *Posit.* 5.) auferatur E A diameter Lunæ, ex *S* A semidiametro *S*olis 9.$\frac{555}{1000}$, restabit *S E*, 8.$\frac{555}{1000}$, & quoniam *S* T est 19.$\frac{11}{100}$ quarum T L est 1 erit *S* L 20.$\frac{11}{100}$, & T L 1. Quapropter Ut *S* L 20.$\frac{11}{100}$ ad T L 1 : :



ita *S* E 8.$\frac{555}{1000}$ ad T G 0.$\frac{4254}{10000}$, adeoque diameter 2.$\frac{85}{100}$ qualium diameter *S*olis est 19.$\frac{11}{100}$. At vero 19.$\frac{11}{100}$ sunt ad 2.$\frac{85}{100}$ prout 382, ad 56.$\frac{97}{100}$, id est 57 ferè.

PROPOSITIO. XVI.

SOl ad Terram est, Ut 55742968 cubus diametri suæ, ad 185193 cubum diametri Terræ, id est, Ut 301 ad 1.

PROPOSITIO. XVII.

Diameter Terræ ad diametrum Lunæ est, Ut 57 ad 20. Nam qualium *S*olis diameter est 19.$\frac{11}{100}$ talium Lunæ est 1 per *Propos.* 9. & qualium idem *S*ol est 19.$\frac{1}{100}$, talium Terra est 2.$\frac{85}{100}$ per *Propos.* 15. Ergo in eisdem partibus Terræ & Lunæ semidiametri sunt, Ut 2.$\frac{85}{100}$ vel 2.$\frac{17}{20}$ ad 1, hoc est, ut 57 ad 20.

P R O-

PROPOSITIO. XVIII.

TErra ad Lunam est, Ut 185193, ad 8000, id est fere $23.\frac{1}{[illegible]}$ pla. sunt etenim, Ut diametrorum cubi, at diametrorum $\left\{\begin{smallmatrix}57\\20\end{smallmatrix}\right.$ cubi sunt $\left\{\begin{smallmatrix}185193\\8000\end{smallmatrix}\right.$ quorum proportio est $23.\frac{1}{[illegible]}$ plani circiter. Ergo

Propositiones hasce nostro modo demonstravimus ex Thesibus *Aristarchi*, beneficio Canonum, *Sin.* Tang. & *Secantium*, qui quidem Canones Authoris tempore non erant in usu. Unde etiam & terminos quantitatum præcise (ex datis) figere non potuit, sed inter binos plerunque statuere coactus est. *Ingeniosissime* tamen demonstrat & istas, & istis subservientes quas (cum usui nobis non sunt) in hac Epitome omisi. Vixit inter *Pithagoram*, & *Archimedem* 280 annis ante *Christum*. Hunc librum *Schickardus* non vidit.

F I N I S.

LEMMATA ARCHIMEDIS,

APUD

GRÆCOS & LATINOS

jam pridem desiderata,

E VETUSTO CODICE M. S. ARABICO.

à JOHANNE GRAVIO TRADUCTA;

Et nunc primùm

CUM ARABUM SCHOLIIS PUBLICATA.

Revisa & pluribus mendis repurgata

à SAMUELE FOSTER.

LONDINI,
Ex Officina LEYBOURNIANA.
M. DC. LIX.